The Effects of Microtargeting on Conversion Rates and the Role of Information Asymmetry

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Abstract

Behavioral advertising targets consumers based on their personal characteristics, while contextual advertising is based on the content consumers are currently viewing. Nowadays, many websites employ a combination of the two, known as *microtargeting*, to display more tailored ads to consumers. Although it is debated whether using behavioral data benefits or harms publishers' revenues, the prevailing belief has been that more personalized ads always yield higher conversion rates. However, recent empirical evidence indicates that less targeted ads can sometimes achieve equal or even greater conversion rates.

We introduce a game-theoretic model to investigate this phenomenon and explain why disabling microtargeted behavioral advertising can sometimes result in increased conversion rates and publisher revenue simultaneously. We link this outcome to the information asymmetry present in the online advertising market. Allowing third-party cookies on a website enables some advertisers to access behavioral data from large ad exchanges, a privilege not universally available. Moreover, not all advertisers have the advanced tools necessary to leverage this additional information effectively. We show that such information asymmetry can reduce market efficiency. Surprisingly, disabling microtargeting can create a more efficient market by providing equal opportunities to all advertisers, despite them being less informed.

Keywords: Online advertising; Targeting; Information asymmetry; Conversion rate; Ad revenue; Auction theory.

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1 Introduction

Can less precise targeting improve ad performance? According to the Dutch public broadcaster NPO,[1] it can. When in January 2020 NPO stopped using tracking cookies on its websites, it saw a dramatic increase in its advertising revenue (Edelman, 2020). Even more surprisingly, in an experiment conducted by NPO with several advertisers, they found that the conversion rate of their ads, one of the most important advertising metrics, not only did not decline, but it was sometimes even better when they targeted visitors with just contextual instead of microtargeted ads (Snelders et al., 2020).

Contextual ads are online advertisements that are displayed to a consumer based on the content of the webpage they are currently visiting. In contrast, microtargeted ads utilize additional behavioral characteristics of the consumer, such as their browsing history, demographics, and interests, to tailor ads specifically to that individual. While contextual ads rely solely on the page's content, microtargeted ads use a more personalized approach that takes into account the user's behavior and preferences. Big ad exchanges are able to collect behavioral data from consumers with the use of third-party cookies that allow them to track consumers across different websites. Advertisers who participate in these ad exchanges can then use this extra information to micro-target consumers with their ads. A publisher can also participate in these ad exchanges by allowing the use of third-party cookies on its website. The main intuition behind this practice is that personalized ads are expected to perform better, leading to an increase in advertisers' willingness to pay for each impression, and as a result the publisher will benefit.

Whether targeted ads can increase publishers' revenue has been a long-standing topic of discussion within the advertising industry. In a study (Ravichandran and Korula, 2019) Google found that disabling third-party cookies decreased publishers' revenue by 52%, favoring microtargeting. In contrast, the NPO case is not the only evidence against microtargeting. In 2018, The New York Times also stopped offering behavioral targeting on its pages experiencing similar outcomes (Davies, 2019).

One prevailing explanation for the increase in publishers' revenue after disabling microtargeting comes from the role of intermediaries. Ad exchanges that offer microtargeting technology do not

[1] The acronym 'NPO' is derived from the Dutch 'Nederlandse Publieke Omroep'.

do it for free. In exchange for their services, exchanges get a big cut of the resulting revenue (Hsiao, 2020). Without the need for microtargeting, publishers can cut off the middlemen and get a larger percentage of advertising revenue. However, this reasoning does not explain the increase in conversion rates observed by NPO. In fact, by removing the middlemen consumers would get less targeted ads, and this should result in fewer conversions, even if the publisher makes more money.

Another possible explanation for how publishers can benefit by sharing less information with advertisers can be found in the academic literature. When advertisers have more information about an ad impression, the market can become thinner, resulting in lower competition and prices (Levin and Milgrom, 2010). However, this explanation does not account for the observed increase in conversion rates either. In fact, although hiding information from the market can lead to an upsurge in competition and prices, it also typically results in reduced market efficiency and, as a consequence, lower conversion rates.[2]

As described above, recent empirical evidence suggests that it is possible to increase revenue and conversion rate simultaneously, while disabling microtargeting. This raises an interesting question about when and why this phenomenon can occur. In this paper, we explore this phenomenon by linking it to the information asymmetry that exists in the advertising market. In practice, not all advertisers have equal access to or can effectively use behavioral data. Larger ad exchanges, such as Google's, may provide more information and better tools to their advertisers compared to what advertisers in smaller exchanges or advertisers who bid directly to a publisher have. This can cause inefficiencies in the market, as less-informed advertisers may underbid, even when their valuation is high, to avoid a winner's curse (McAfee and McMillan, 1987). It may seem that turning off behavioral advertising and reducing information in the market would increase inefficiencies, leading to lower conversion rates and revenues.[3] However, we demonstrate that disabling microtargeting can increase market efficiency by creating fairer competition and limiting underbidding behavior.

[2]This is because the advertiser who ends up getting the ad impression is less likely to be the advertiser who values it the most.

[3]The intuition for a potential lower conversion rate is that by hiding information from bidders, it is less likely that the bidder with the highest valuation will win. The intuition regarding revenue loss is related to the linkage principle (Milgrom and Weber, 1982); revenue would be expected to decrease because more bidders would underbid to avoid the winner's curse.

Contributions

One of our central results is that, under specific conditions, a publisher can improve their revenue and the conversion rate simultaneously by disabling behavioral targeting (Proposition 1). Two conditions must be met for this outcome to occur. The first condition is a positive correlation among the advertisers' valuations linked to the consumer's behavioral data. For example, if a consumer's behavioral data indicates high spending and high engagement with ads, this information could suggest high valuation for most advertisers, thus fulfilling the first condition because of the correlated valuations. The second necessary condition requires that not all advertisers have access to the consumer's behavioral information, thereby creating information asymmetry among advertisers. If either of these conditions is unmet, the result of Proposition 1 does not hold. More specifically, when behavioral values are independent, the publisher's revenue can sometimes increase and sometimes decrease upon disabling behavioral targeting (Figure 8), but the conversion rate remains the same or decreases (Proposition 3). When there is no information asymmetry, neither the revenue nor the conversion rate can be improved by disabling behavioral targeting (Proposition 2). Finally, when there is no information asymmetry and behavioral values are independent, disabling behavioral targeting may cause an increase or decrease in revenue (Proposition 9), but the conversion rate never improves (Proposition 3). Figure 13 provides a concise summary of our main results.

The intuition behind our main finding is as follows. In a market with information asymmetry and correlated advertiser valuations, less-informed advertisers often resort to underbidding or overbidding. Sometimes they bid conservatively (underbid) to avoid overpaying for impressions of limited value, and sometimes they bid aggresively (overbid) to increase their probability of securing valuable impressions against better-informed advertisers. Both of these bidding behaviors can lead to an inefficient market, where the advertiser who wins the impression is not necessarily the one who values it the most. At first glance, preventing behavioral targeting by all advertisers could appear counterproductive, as it results in more uninformed advertisers. However, a key advantage is that now all advertisers are equally informed. This eliminates information asymmetry and causes more efficient bidding behavior, leading to a more efficient market. The winner is now more likely to be the advertiser who values the impression the most, despite their lack of precise valuation knowledge.

Our findings have several implications for publishers, advertisers, and consumers. From the perspective of publishers, it is crucial to carefully consider the effects of information asymmetry in advertising auctions. If they cannot provide all available information to every advertiser, it might be more advantageous to disable behavioral targeting, restrict the information available to all advertisers, and establish a more equitable environment with more efficient outcomes. For advertisers, our findings suggest that more personalized ads do not necessarily result in more conversions. Additionally, regulations related to data privacy and consumer protection do not inherently disadvantage them. Lastly, for consumers, the move by publishers towards more privacy-friendly advertising not only promises the obvious benefit of enhanced privacy but also potentially offers more relevant ads. Our findings suggest that disabling microtargeting can improve the overall ad ecosystem, thus benefiting consumers.

2 Related Literature

This research contributes to the growing literature on targeted advertising and online advertising auctions (e.g., Roy, 2000; Esteban et al., 2001; Gal-Or et al., 2006; Dong et al., 2009; Athey and Gans, 2010; Goldfarb and Tucker, 2011; Tucker, 2014; Chandra and Kaiser, 2014; Summers et al., 2016; Trusov et al., 2016; Lee et al., 2018; Sayedi, 2018; Deng and Mela, 2018; Choi and Sayedi, 2019; Zia and Rao, 2019; Choi et al., 2020; Gordon et al., 2021; Despotakis et al., 2021; Shin and Yu, 2021; Long et al., 2022; Choi et al., 2023). Below, we discuss in more detail some related theory papers on targeted advertising.

From the advertisers' perspective, improving firms' ability to target consumers typically improves revenues, but in some cases it can have negative effects. For example, Chen et al. (2001) study the effects of imperfect targetability on prices for different segments of consumers. Interestingly, they found that improving the targetability of a firm can sometimes benefit both the firm and its competitor. Iyer et al. (2005) describe a model of competing firms who can target different segments of consumers with advertising and show that targeted advertising will improve the firms' profits and, moreover, it can sometimes be more valuable than targeted pricing. Bergemann and Bonatti (2011) show that better targeting causes an increase in the number of consumer-product matches, but prices of ads change non-monotonically in the targeting capacity. Brahim et al. (2011)

study a model with two firms competing in prices and targeted advertising. They show that firms' profits can be lower with targeted relative to random advertising. Esteves and Resende (2016) study how targeted advertising can be used by competing firms to price discriminate different segments of consumers. Zhang and Katona (2012) study how contextual advertising affects product market competition. Johnson (2013) considers targeted advertising in combination with advertising avoidance technology. He shows that targeting will increase firms' profits, but it can make consumers worse off. Hummel and McAfee (2016) study how the number of bidders in an auction affects a seller's revenue under two different settings (bundling vs. targeting). A difference in our model is that bidders' valuations need not be independent. Despotakis and Yu (2022) study a multidimensional targeting model and show that sometimes the use of multiple dimensions of data to target consumers can have negative effects for a firm.

De Corniere and De Nijs (2016) show that when a platform chooses to reveal the information it has about a consumer to advertisers, the advertisers will set higher prices in anticipation of a better matching. This will benefit the advertisers and the platform. Our setting differs in that revealing information can actually worsen the matching between the advertisers and the consumer, resulting in a lower social welfare. This is because in addition to the full disclosure or non-disclosure of information, we also consider the case where not all advertisers have access to the same information about the consumer, and this asymmetry plays an important role in our model. Shen and Miguel Villas-Boas (2018) study advertising based on the past purchase behavior of consumers and examine how it affects product prices for a monopolist. Rafieian and Yoganarasimhan (2021) show that the revenues of ad-networks can increase when they allow users to preserve their privacy. This is because more precise targeting can thin out the market and soften competition, in a similar fashion to Levin and Milgrom (2010). However, when this happens, the targeting becomes less efficient. In our model, we can replicate this effect for the case of independent valuations between the advertisers, but for the case of dependent valuations, we see that revenue and efficiency can increase or decrease together.

Ada et al. (2022) study the impact of providing ad context information in ad exchanges and find that in most cases ad exchanges can boost publishers' revenues by sharing context information with ad buyers. (Shin and Shin, 2022) demonstrate that irrelevant advertising can stem from strategic decisions within the ad agency-advertiser relationship, rather than simply technological imperfections. The study also explores how contractual restrictions can lead to inefficiencies in ad delivery, and suggests that the prevalence of irrelevant ads may decrease, but not disappear, as the number of impressions available in the market increases. Choi and Sayedi (2023b) investigate the effects of private exchanges on the display advertising market, finding that while private exchanges offer higher quality impressions compared to open exchanges, they can also create information asymmetry among advertisers, which can hurt publisher's revenue. Choi and Sayedi (2023a) examine the effects of ad agencies on the online advertising market, revealing that publishers face a trade-off when deciding whether to withhold targeting information from agencies, which can either mitigate "bid rotation" and attract direct advertisers or reduce the efficiency of allocation for agency-using advertisers.

This paper contributes to the targeted advertising literature by examining the role of information asymmetry in targeted advertising. In the presence of this asymmetry, we show that both publisher revenue and conversion rates can increase simultaneously when the publisher disables microtargeting, which is not the case for symmetric advertisers. It also contributes to the literature on online advertising auctions. In our model we examine the interaction between different conditions on advertisers' valuations (correlated and independent), which consist of two different components (contextual and behavioral), and different information settings (where different advertisers have access to different information), providing a comprehensive look at how information asymmetries can affect the equilibrium market outcomes.

3 Model

Two advertisers are competing for a single impression, provided by a publisher. Advertiser 1 places their bid for the impression through an ad exchange (hereafter referred to as the *exchange advertiser*), while Advertiser 2 submits their bid directly to the publisher (hereafter referred to as the *direct advertiser*).[4] The exchange advertiser has access to the behavioral data of the consumer generating the impression, as this information can be provided by the exchange. The direct advertiser,

[4]We have opted to present the simplest version of the model with two advertisers here, which is sufficient to explain the core insights and intuitions in Section 4. In Sections 5 and 6, we extend the model to $n \geq 2$ advertisers, with a subset of $n_1 \leq n$ exchange advertisers and a subset of $n_2 = n - n_1$ direct advertisers. We show that our central findings are robust to changes in the number of advertisers, and the insights derived from the main model in Section 4 remain consistent.

however, lacks access to this data. Both advertisers know the contextual data associated with the impression.

Advertiser i's valuation for the impression is $v_i = \kappa \cdot b_i + (1-\kappa) \cdot c_i$, where b_i and c_i are random variables in $[0,1]$ that correspond to the behavioral data and the contextulal data of the consumer, respectively. These values capture how well the consumer data matches each advertiser, where a higher value indicates a better match.[5] The parameter $\kappa \in [0,1]$ allows for different contributions of the behavioral and the contextual data in the advertisers' valuations. The larger the parameter κ is, the more important the behavioral data is in comparison to the contextual data.[6] We can interpret the valuation $v_i \in [0,1]$ as the probability that the impression will result in a conversion, i.e. an advertiser receives a normalized revenue of 1 for each conversion.

For a random impression, we assume that the values c_i are i.i.d. draws from a uniform distribution G in $[0,1]$.[7] Intuitively, the assumption of independence means that context by itself does not tell us anything about the valuation of an unknown advertiser. For example, if a consumer is reading an article about cars, car advertisers may have a high valuation for the consumer, while restaurant advertisers may have a lower valuation (and it is not, for example, the case that every advertiser has a high valuation).[8]

However, we allow the random variables b_i to be potentially correlated with each other. For example, a consumer with high income who is a high spender might be desirable by all advertisers, independently of the advertised product. But it is also possible for the b_i's to be independent, as it would be more appropriate for e.g. data related to past browser history. To capture these two scenarios, we consider the following two extreme cases:

- Common-value case (CV): $b_1 = b_2 =: b$, where b is drawn from distribution F.
- Independent-values case (IV): b_1, b_2 are i.i.d. draws from distribution F.

[5] Note that an advertiser with no access to behavioral data will have an expected valuation $\mathbb{E}[v_i] = \kappa \cdot \mathbb{E}[b_i] + (1-\kappa) \cdot c_i$, therefore if they learn b_i, their posterior impression valuation can be higher or lower than their prior.

[6] For a significant part of the analysis in the paper, we will focus on the case where $\kappa \geq 1/2$, which is the most interesting one. When κ is small, the behavioral values are less important and as a result the impact of the asymmetry between the exchange and the direct advertisers is smaller.

[7] In Sections 5 and 6, we consider an arbitrary distribution G instead of a uniform one and show that the main results remain robust.

[8] The assumption of independent contextual random variables is not necessary for the results; it is made to simplify the model, in line with the standard model of private-value auctions (see e.g., Krishna, 2009). The intent is to concentrate on the different possible cases (independent values vs. common value) for the behavioral random variables, which are the variables for which information asymmetry can occur in our context.

For parsimony and analytical tractability, we model the distribution F as a Bernoulli distribution in $\{0, 1\}$, with $\Pr[b_i = 1] = p$ for some probability $p \in [0, 1]$.

The impression is sold using a second-price auction run by the publisher. This means that the advertiser with the highest bid wins and pays the second-highest bid to the publisher.[9] Finally, to study the effect of microtargeting with behavioral data on the equilibrium outcome, we consider and compare three settings. In the first one, called IA (short for microtargeting with Information Asymmetry, where both contextual and behavioral data are used), the publisher allows the use of behavioral advertising and everything is like in the model described above. In the second setting, called CT (short for only Contextual Targeting), the publisher prevents the use of behavioral advertising and none of the advertisers have access to behavioral data.[10] As an additional benchmark and for completeness we also consider a third setting, called FI (short for Full Information), where all advertisers have access to all the data. This third setting eliminates the information asymmetry from the market, similarly to the CT setting, nevertheless, we show that the market outcomes can sometimes differ. Note that microtargeting also occurs in the FI setting, but without information asymmetry.

For each behavioral setting $\sigma \in \{\mathrm{CV}, \mathrm{IV}\}$ (common-value, independent-values) and each information setting $\tau \in \{\mathrm{FI}, \mathrm{IA}, \mathrm{CT}\}$ we denote by W_τ^σ and V_τ^σ the publisher's expected revenue and the expected conversion rate, respectively. Similarly, we denote by E_τ^σ and D_τ^σ the exchange advertiser's and the direct advertiser's expected payoffs, respectively. Table 1 summarizes all the notation described above.

The rest of the paper is structured as follows. In Section 4, we present the main results and insights for the main model described above. Then, in Sections 5 and 6 we show the robustness of our results in more general settings that include a larger number of advertisers and arbitrary

[9] In practice, more elaborate selling mechanisms are possible. For example, the ad exchange can run its own auction among its advertisers and submit the clearing price to the publisher; the publisher can then run an additional auction with all the received bids to determine the winner. Each auction can also be of different formats, e.g. second-price or first-price. Here, we abstract away from the complications of the selling mechanism itself by using a standard second-price auction for selling the impression. Even though alternative mechanisms can complicate the analysis significantly, the intuition for the results described in the paper still holds. To demonstrate this, in Lemma 4 we show that one of our main results, that the conversion rate can increase by disabling behavioral targeting, holds for a wide variety of selling mechanisms, including e.g. a single or multiple first-price auctions. For completeness, in Section A.3 we show that for first-price auctions the publisher's revenue can also increase by disabling behavioral targeting (Proposition 11). For other selling mechanisms, similar intuition applies.

[10] Since the only value of the ad exchange in our model is the access it gives to behavioral data when the publisher allows the use of third-party cookies, the CT setting is similar to removing the ad exchange from the model and having all advertisers bidding directly to the publisher.

distributions G. In Section 5 we do it analytically for the results where a proof is feasible despite the lack of a closed-form bidding function.[11] In Section 6 we start by showing the existence of a pure symmetric equilibrium for the general model;[12] we then numerically approximate the bidding function for some general examples and show the robustness of the results for the general case. All proofs are relegated to Appendix A.1 and a summary of all key formulas can be found in Appendix A.2.

4 Analysis and Main Intuitions

In this section, we start with the main model with two advertisers. In subsection 4.1 we consider the common-value case (CV) and in subsection 4.2 we consider the independent-values case (IV). In subsection 4.3 we compare and discuss the differences between the common-value and independent-values cases in terms of publisher's revenue, conversion rates, and advertisers' payoffs.

4.1 Common-value case

Under the common-value case in the CT and FI settings, both advertisers will have the same information, therefore, in the second-price auction they will truthfully bid their valuation (in FI) or their expected valuation (in CT, where they do not know the actual valuation) (see e.g., Krishna, 2009). However, in the IA setting the exchange advertiser is more informed than the direct advertiser. As a consequence of this asymmetry, the exchange advertiser will still bid their true valuation, but the direct advertiser might not always do that. We start off with Lemma 1 on the bidding function of the direct advertiser.

Lemma 1 (Advertisers' bidding behavior)**.** *Under the common-value* IA *setting, the exchange advertiser bids their true valuation, while given contextual value $c \in [0, 1]$ the direct advertiser's*

[11] More specifically, the bidding function of the direct advertisers is the solution to a differential equation that does not always have a closed-form solution (see equation 5 and the proof of Lemma 3).

[12] More specifically, we show that there is a pure symmetric equilibrium bidding strategy for the direct advertisers under the common-value IA setting.

Information settings	
FI	Full Information. The publisher provides the behavioral data to all the advertisers.
IA	Microtargeting with Information Asymmetry. Only the exchange advertisers have access to behavioral data.
CT	Contextual Targeting. No advertiser has access to behavioral data.
Behavioral-value settings	
CV	Common Value. The behavioral value is the same for all the advertisers.
IV	Independent Values. The behavioral values are independent between advertisers.
Parameters	
κ	Behavioral Weight. The importance of behavioral data compared to contextual data on the advertisers' valuations.
p	Behavioral Probability. The probability that the behavioral value b of a random consumer is high for an advertiser.
Market Metrics	
V_τ^σ	Expected conversion rate under the behavioral-value setting $\sigma \in \{\text{CV}, \text{IV}\}$ and the information setting $\tau \in \{\text{FI}, \text{IA}, \text{CT}\}$.
W_τ^σ	Publisher's expected revenue under the behavioral-value setting $\sigma \in \{\text{CV}, \text{IV}\}$ and the information setting $\tau \in \{\text{FI}, \text{IA}, \text{CT}\}$.
E_τ^σ	Exchange advertiser's expected payoff under the behavioral-value setting $\sigma \in \{\text{CV}, \text{IV}\}$ and the information setting $\tau \in \{\text{FI}, \text{IA}, \text{CT}\}$.
D_τ^σ	Direct advertiser's expected payoff under the behavioral-value setting $\sigma \in \{\text{CV}, \text{IV}\}$ and the information setting $\tau \in \{\text{FI}, \text{IA}, \text{CT}\}$.
Others	
$v = \kappa b + (1-\kappa)c$	Advertiser's valuation for behavioral value b and contextual value c. It is also used as a proxy for conversion rate.
$\beta(c)$	Equilibrium bidding function of a direct advertiser under the common-value IA setting, where the direct advertiser does not know b but they know c.
Generalizations	
Number of advertisers	
n_1	Number of exchange advertisers. They have access to behavioral data, except in the contextual-targeting information setting (CT).
n_2	Number of direct advertisers. They only have access to contextual data, except in the full-information setting (FI).
$n = n_1 + n_2$	Total number of advertisers.
Distributions	
G	Contextual Distribution. The CDF of the distribution of the contextual value of a random consumer for an advertiser.

Table 1: Summary of Notation

bidding function is

$$\beta(c) := \begin{cases} (1-\kappa)c, & \textit{if } 0 \le c < \min\left\{\frac{\sqrt{1-p}}{\sqrt{1-p}+\sqrt{p}}, \frac{\sqrt{1-p}\kappa}{1-\kappa}\right\}, \\ \kappa p + (1-\kappa)c, & \textit{if } \min\left\{\frac{\sqrt{1-p}}{\sqrt{1-p}+\sqrt{p}}, \frac{\sqrt{1-p}\kappa}{1-\kappa}\right\} \le c < \max\left\{\frac{\sqrt{1-p}}{\sqrt{1-p}+\sqrt{p}}, 1-\frac{\sqrt{p}\kappa}{1-\kappa}\right\}, \\ \kappa + (1-\kappa)c, & \textit{if } \max\left\{\frac{\sqrt{1-p}}{\sqrt{1-p}+\sqrt{p}}, 1-\frac{\sqrt{p}\kappa}{1-\kappa}\right\} \le c \le 1. \end{cases} \tag{1}$$

The intuition behind Lemma 1 is the following. If the contextual value c of the direct advertiser is relatively low, they bid as if the common behavioral value b is 0. This is because if they assume some other value $b = x > 0$, they risk overpaying for the impression in the case where $b = 0$ and $(1-\kappa)\cdot c < (1-\kappa)\cdot c' < \kappa \cdot x + (1-\kappa)\cdot c$ (where c' is the contextual value of the exchange advertiser), where they end up with a negative payoff of $(1-\kappa)\cdot(c-c')$. When $c < \min\left\{\frac{\sqrt{1-p}}{\sqrt{1-p}+\sqrt{p}}, \frac{\sqrt{1-p}\kappa}{1-\kappa}\right\}$, this risk is too high to take. However, when the contextual value c is high $\left(c \ge \max\left\{\frac{\sqrt{1-p}}{\sqrt{1-p}+\sqrt{p}}, 1-\frac{\sqrt{p}\kappa}{1-\kappa}\right\}\right)$, it is very likely that $c > c'$, therefore they are not afraid to bid as if $b = 1$, because they have a higher incentive to win and avoid losing impressions with good behavioral values. For medium values of c, both the risks of overpaying for a bad impression and losing a good impression are too high to make any assumption about b, therefore the advertiser simply bids their expected valuation (note that the expected value of b is p).

Note that as κ increases, i.e. as the behavioral data becomes more important, the middle interval of c where the direct advertiser bids their expected valuation shrinks, and for $\kappa \ge 1/2$ it disappears, i.e. the advertiser either underbids or overbids depending on c (see also Figure 1 where the bidding function is shown for different values of κ). Since the role of information asymmetry is more important for larger values of κ and it is where the more interesting results occur, for simplicity for some of the analysis, we focus on the case where $\kappa \ge 1/2$, unless otherwise noted.

Note also that when the value of p is low, the region of c where the underbidding occurs is wider compared to the overbidding region, but the amount of underbidding (κp) is smaller compared to the amount of overbidding ($\kappa(1-p)$). On the other hand, when p is high, overbidding is more common but the amount of overbidding is lower. This is illustrated in Figure 2.

In Lemma 2 of Section 5 we show a generalization of Lemma 1 for any $n_1 \ge 1$, any distribution G, $p \in [0,1]$, and $\kappa \ge 1/2$. Lemma 3 in Section 6 is a further generalization for the more general case with $n_2 \ge 1$ (where the bidding function does not always have a closed-form expression). The

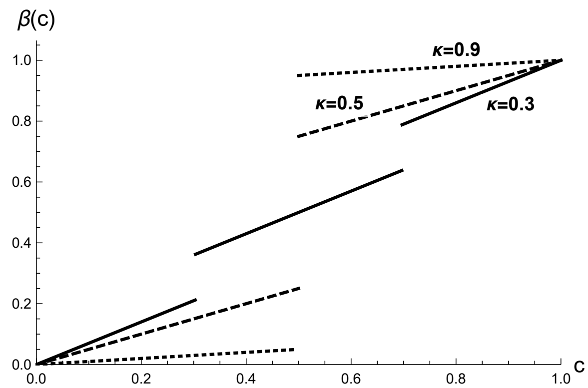
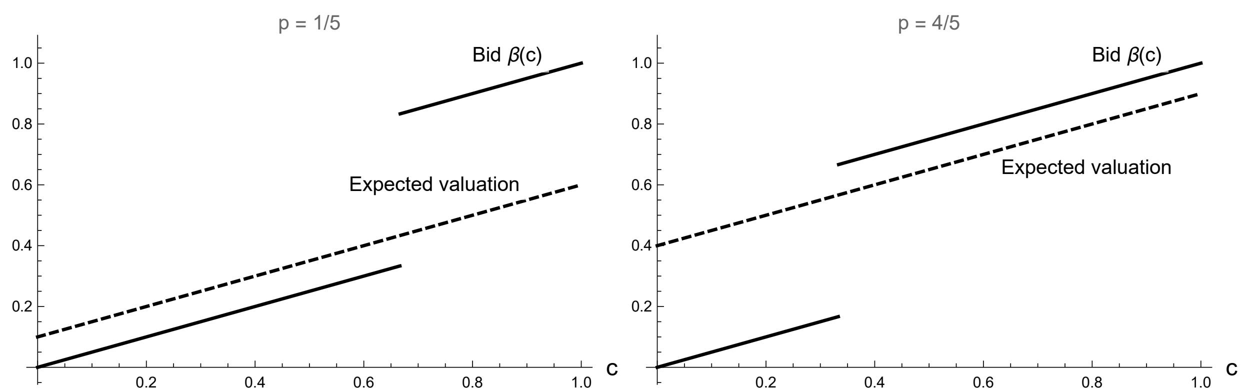


Figure 1: Bidding function of the direct advertiser for different values of κ (solid line for $\kappa = 0.3$, dashed line for $\kappa = 0.5$, and dotted line for $\kappa = 0.9$), $n_1 = n_2 = 1$, $p = 1/2$, and $G(x) = x$. Notice that for large contextual values c, as κ increases there is more overbidding, while for small contextual values c, as κ increases there is more underbidding.

same intuition as for Lemma 1 applies to Lemma 2 and Lemma 3 as well.



(a) $p = 1/5$.

(b) $p = 4/5$.

Figure 2: Bidding function of the direct advertiser (solid line) compared to their expected valuation (dashed line), for different values of p, $n_1 = n_2 = 1$, $\kappa = 1/2$, and $G(x) = x$. Notice that for small values of p (left) the region of overbidding is smaller than the region of underbidding, but the amount of overbidding ($\kappa(1-p)$) is larger than the amount of underbidding (κp). For large values of p (right) the opposite happens.

The bidding function of Lemma 1 sometimes results in an inefficient market under the IA information setting. More specifically, both the underbidding and the overbidding can result in lower conversion rate compared to the CT setting (where every advertiser bids their expected valuation). This is illustrated in Example 1.

Example 1 (Inefficiency of non-truthful bidding)**.** Let $\kappa = p = 1/2$. Then the direct advertiser bids $c/2$ if $c < 1/2$ and $(1+c)/2$ if $c \geq 1/2$, where c is their contextual value. The following two examples illustrate the inefficiency caused by the non-truthful bidding of the direct advertiser. They show that both underbidding and overbidding can result in lower conversion rates.

- **Inefficiency of underbidding**

  (IA setting) Suppose that the common behavioral value is high, i.e. $b = 1$, the exchange advertiser has contextual value $c_1 = 1/6$, and the direct advertiser has contextual value $c_2 = 1/3$. The actual valuations of the two advertisers are $v_1 = (1 + c_1)/2 = 7/12$ for the exchange advertiser and $v_2 = (1 + c_2)/2 = 8/12$ for the direct advertiser. The exchange advertiser bids their actual valuation $\beta_1 = 7/12$, but the direct advertiser underbids, i.e. $\beta_2 = c_2/2 = 2/12$. As a result, the direct advertiser loses the auction even though they have a higher valuation. Thus, the conversion rate ends up being lower than what it could be in a more efficient auction.

  (CT setting) If none of the advertisers knew the behavioral value b, then both advertisers would bid their expected valuations, i.e. $\beta_1 = 1/4 + c_1/2 = 4/12$ and $\beta_2 = 1/4 + c_2/2 = 5/12$. Thus, the advertiser with the highest valuation would win, leading to a higher conversion rate.

- **Inefficiency of overbidding**

  (IA setting) Suppose that the common behavioral value is low, i.e. $b = 0$, the exchange advertiser has contextual value $c_1 = 5/6$, and the direct advertiser has contextual value $c_2 = 2/3$. The actual valuations of the two advertisers are $v_1 = c_1/2 = 5/12$ for the exchange advertiser and $v_2 = c_2/2 = 4/12$ for the direct advertiser. The exchange advertiser bids their actual valuation $\beta_1 = 5/12$, but the direct advertiser overbids, i.e. $\beta_2 = (1 + c_2)/2 = 10/12$. As a result, the direct advertiser wins the auction even though they have a lower valuation. Thus, the conversion rate ends up being lower than what it could be in a more efficient auction.

  (CT setting) If none of the advertisers knew the behavioral value b, then both advertisers would bid their expected valuations, i.e. $\beta_1 = 1/4 + c_1/2 = 8/12$ and $\beta_2 = 1/4 + c_2/2 = 7/12$. Thus, the advertiser with the highest valuation would win, leading to a higher conversion rate.

As we can see in Example 1, there are cases where under the IA setting the advertiser with the highest valuation does not win, either due to the underbidding or due to the overbidding of the direct advertiser. In contrast, under the CT setting, the highest-valuation advertiser always wins, because every bidder bids their expected valuation, and the winner is determined based on the contextual values. This results in a higher conversion rate for the CT setting, as shown in Proposition 1.

For the publisher's revenue, things are less clear. On the one hand, the underbidding that occurs under IA can hurt the publisher, but on the other hand, the overbidding can benefit the publisher because it can increase the prices. Surprisingly, the opposite can happen too; underbidding can sometimes increase publisher's revenue, and overbidding can decrease it, as illustrated in Example 2.

Example 2 (The effects of non-truthful bidding on revenue)**.** Let $\kappa = 1/2$ and $p = 1/3$. Then the direct advertiser bids $c/2$ if $c < 2 - \sqrt{2}$ and $(1 + c)/2$ if $c \geq 2 - \sqrt{2}$, where c is their contextual value. The following two examples illustrate that, counter-intuitively, underbidding can sometimes increase publisher's revenue, and overbidding can sometimes decrease it.

- **Underbidding can increase publisher's revenue**

  (IA setting) Suppose that the common behavioral value is high, i.e. $b = 1$, the exchange advertiser has contextual value $c_1 = 1/12$, and the direct advertiser has contextual value $c_2 = 1/2$. The actual valuations of the two advertisers are $v_1 = (1 + c_1)/2 = 13/24$ for the exchange advertiser and $v_2 = (1 + c_2)/2 = 18/24$ for the direct advertiser. The exchange advertiser bids their actual valuation $\beta_1 = 13/24$, but the direct advertiser underbids, i.e. $\beta_2 = c_2/2 = 6/24$. The exchange advertiser wins and pays β_2, therefore, the publisher's revenue is $6/24$.

  (CT setting) If none of the advertisers knew the behavioral value b, then both advertisers would bid their expected valuations, i.e. $\beta_1 = 1/6 + c_1/2 = 5/24$ and $\beta_2 = 1/6 + c_2/2 = 10/24$. Then the direct advertiser would win and pay β_1. Therefore, publisher's revenue would be $5/24$, which is lower than the revenue under the IA setting.

- **Overbidding can decrease publisher's revenue**

  (IA setting) Suppose that the common behavioral value is low, i.e. $b = 0$, the exchange advertiser has contextual value $c_1 = 5/6$, and the direct advertiser has contextual value $c_2 = 3/4$. The actual valuations of the two advertisers are $v_1 = c_1/2 = 10/24$ for the exchange advertiser and $v_2 = c_2/2 = 9/24$ for the direct advertiser. The exchange advertiser bids their actual valuation $\beta_1 = 10/24$, but the direct advertiser overbids, i.e. $\beta_2 = (1 + c_2)/2 = 21/24$. The direct advertiser wins and pays β_1, therefore, publisher's revenue is $10/24$.

(CT setting) If none of the advertisers knew the behavioral value b, then both advertisers would bid their expected valuations, i.e. $\beta_1 = 1/6 + c_1/2 = 14/24$ and $\beta_2 = 1/6 + c_2/2 = 13/24$. Then the exchange advertiser would win and pay β_2. Therefore, publisher's revenue would be $13/24$, which is higher than the revenue under the IA setting.

Despite valuation instances like those in Example 2, in Proposition 1 we show that the overall publisher's expected revenue is higher under the CT setting.

Proposition 1 (Common-value and information asymmetry)**.** *Under the common-value behavioral setting, the publisher can improve both the conversion rate and the expected revenue if it hides the behavioral information from all the advertisers. In other words, we have* $V_{\mathrm{IA}}^{\mathrm{CV}} \leq V_{\mathrm{CT}}^{\mathrm{CV}}$ *and* $W_{\mathrm{IA}}^{\mathrm{CV}} \leq W_{\mathrm{CT}}^{\mathrm{CV}}$*.*

Proposition 1 shows that if the publisher has some useful information about a consumer but cannot provide this information to all advertisers, it can achieve a higher conversion rate by hiding the information from everyone rather than giving it only to some advertisers. As an added benefit, the publisher can also simultaneously increase its revenue by hiding this information for all advertisers. The main reason this happens is the inefficiency of the non-truthful bidding of the direct advertiser under the IA setting.

Given the result of Proposition 1, one may wonder if the same can happen when there is no information asymmetry between the advertisers. In other words, if all advertisers have access to the same information, is it still possible that less information can simultaneously increase the conversion rate and the publisher's revenue? In Proposition 2 we show that this cannot happen under the common-value behavioral setting.

Proposition 2 (Common-value and full information)**.** *Under the common-value behavioral setting, both the conversion rate and the expected revenue remain unchanged when all advertisers have access to the same information (i.e., when all advertisers have access to behavioral data or none of the advertisers have access to behavioral data). In other words, it holds that* $V_{\mathrm{FI}}^{\mathrm{CV}} = V_{\mathrm{CT}}^{\mathrm{CV}}$ *and* $W_{\mathrm{FI}}^{\mathrm{CV}} = W_{\mathrm{CT}}^{\mathrm{CV}}$*.*

The equality $V_{\mathrm{FI}}^{\mathrm{CV}} = V_{\mathrm{CT}}^{\mathrm{CV}}$ is relatively easy to see, whereas the equality $W_{\mathrm{FI}}^{\mathrm{CV}} = W_{\mathrm{CT}}^{\mathrm{CV}}$ is less straightforward. Under the common-value setting, since all advertisers have the same behavioral value, when all have the same information, the behavioral part of their bids is the same for everyone; therefore, the winner of the auction is purely determined by their contextual values in both the FI and the CT settings. As a result, the conversion rate remains unchanged.

For the revenue, when the common behavioral value is low, publisher's revenue is higher under the CT setting because every advertiser bids above their actual valuation. In contrast, when the common behavioral value is low, the publisher's revenue is lower under the CT setting because every advertiser bids below their actual valuation. Due to the linearity of the expectation, the average revenue remains the same in the two information settings.

Note that as we move from the FI to the IA and then to the CT information setting, the overall information to the advertisers is reduced. As a result, the inequality $W_{\mathrm{FI}}^{\mathrm{CV}} \geq W_{\mathrm{IA}}^{\mathrm{CV}}$ agrees with the linkage principle (Milgrom and Weber, 1982) which would suggest that revealing information is better for the revenue, but the inequality $W_{\mathrm{IA}}^{\mathrm{CV}} \leq W_{\mathrm{CT}}^{\mathrm{CV}}$ violates the principle which happens due to the information asymmetry.[13]

In this section, we have seen that under the common-value setting there is a non-monotonic relationship between the amount of information available to the advertisers and the efficiency of the auction; as we reduce the information, efficiency (i.e. the conversion rate) first goes down and then goes up again. We have also seen that a similar effect occurs for the publisher's revenue. In Section 4.2 we show that this is no longer true when the behavioral values are independent.

4.2 Independent-values case

In contrast to the common-value case, when the behavioral values of advertisers are independent, all advertisers will bid truthfully according to their (expected) valuation.

In the independent-values case, the intuitive result that less information to the advertisers decreases the conversion rate now holds. This is still not true for every valuation instance, as illustrated in Example 3, but it is true for the expected conversion rates, as shown in Proposition 3.

Example 3. Let $\kappa = p = 1/2$.

[13] For some other cases where the principle is violated for different reasons, see e.g. Perry and Reny (1999); Fang and Parreiras (2003); Krishna (2009); Despotakis et al. (2017).

(IA setting) Suppose that the exchange advertiser has a behavioral value $b_1 = 1$ and a contextual value $c_1 = 3/8$, and the direct advertiser has a behavioral value $b_2 = 1$ and a contextual value $c_2 = 5/8$. The actual valuations of the two advertisers are $v_1 = (1 + c_1)/2 = 11/16$ for the exchange advertiser and $v_2 = (1 + c_2)/2 = 13/16$ for the direct advertiser. The exchange advertiser bids their actual valuation $\beta_1 = 11/16$, but the direct advertiser bids their expected valuation, i.e. $\beta_2 = 1/4 + c_2/2 = 9/16$. As a result, the direct advertiser loses the auction even though they have higher valuation.

(CT setting) If none of the advertisers knew their behavioral values b_i, then both advertisers would bid their expected valuations, i.e. $\beta_1 = 1/4 + c_1/2 = 7/16$ and $\beta_2 = 1/4 + c_2/2 = 9/16$. Thus, the advertiser with the highest valuation would win, leading to a higher conversion rate than the IA setting.

Despite valuation instances like those in Example 3, in Proposition 3 we show that the overall expected conversion rate increases with more information, under the independent-values setting.

Proposition 3 (Independent-values, conversion rates)**.** *Under the independent-values behavioral setting, the less information advertisers have overall, the lower the conversion rate is. More specifically,* $V_{\text{FI}}^{\text{IV}} \geq V_{\text{IA}}^{\text{IV}} \geq V_{\text{CT}}^{\text{IV}}$.

Proposition 3 shows that the dependence between the behavioral values of different advertisers is an essential element for the result of Proposition 1, since for independent values it no longer holds.

With respect to the publisher's revenue, the result is less intuitive. Proposition 4 shows that as we provide more information in general to advertisers, publisher revenue decreases.

Proposition 4 (Independent-values, publisher's revenues)**.** *Under the independent-values behavioral setting, the less information advertisers have overall, the higher publisher's revenue is. More specifically, we have* $W_{\text{FI}}^{\text{IV}} \leq W_{\text{IA}}^{\text{IV}} \leq W_{\text{CT}}^{\text{IV}}$.[14]

The result of Proposition 4 is sensitive to the number of advertisers (in contrast to the previous results that hold for arbitrary number of advertisers; see Section 5). What happens in general

[14] We want to highlight that this result holds for a low number of advertisers (e.g. two, like in the main model), but unlike the other results it does not always generalize for more advertisers. In Section 5.2 we consider the general case and discuss the details on this.

is that, for a small number of advertisers, less information is better, but for a large number of advertisers, more information is better. This is due to a version of the market-thinning effect (Levin and Milgrom, 2010). When there are few advertisers in the market, as they become more informed their values spread out, and there is less competition in the high valuations. But as the number of advertisers becomes larger and the competition increases, more information should improve publisher's revenue. More specifically, when $n_1 = n_2 = 1$ (i.e. there is one advertiser of each type) it holds that more information decreases revenue, but as n_1 and n_2 increase, at some point this stops being true. The exact threshold for the number of advertisers where monotonicity changes depends on the value of p, with a lower p increasing the threshold, the behavioral-value weight κ, with higher κ increasing the threshold, and the contextual distribution G (see Proposition 9 and Figure 7 for more details).[15]

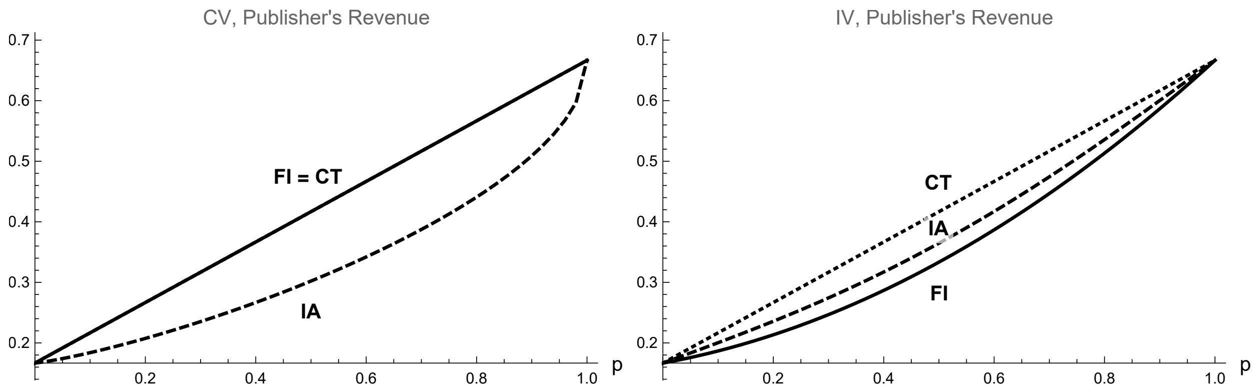
4.3 Comparison of the Behavioral-Value Settings and Advertisers' Payoffs

Now that we have the results for the simple model with two advertisers, we can compare the two behavioral-value settings (the common-value case and the independent-values cases) to each other in terms of their consequences for the publisher's revenue, conversion rates, and advertisers' payoffs.

We start with the publisher's revenue in Figure 3. In the two plots of Figure 3, we see the revenue under the three different information settings as the behavioral probability p changes in $[0, 1]$. In the common-value case in Figure 3(a), we can see that starting from the IA setting and eliminating the information asymmetry by going towards FI or CT, the publisher's revenue increases. This is due to the underbidding and overbidding behavior that occurs in IA, as discussed in Section 4.1. In contrast to Figure 3(a), in the independent-values case in Figure 3(b) we observe a monotonic change in revenue. As we add information to the market (moving from CT to IA and then to FI), publisher's revenue goes down, as described in Proposition 4.

Next, we move to the conversion rates in Figure 4. What we observe in Figure 4(a) is one of our main findings. What happens here is that in the IA setting, according to Lemma 1, a direct advertiser with high valuation often bids conservatively and loses to an exchange advertiser with a lower valuation. In addition, a direct advertiser with low valuation often bids aggressively and wins

[15] It is interesting to note that there are also cases where the expected revenue is non-monotonic with respect to the total amount of information that is available to the advertisers. In other words, all six different orderings of $W_{\mathrm{FI}}^{\mathrm{IV}}$, $W_{\mathrm{IA}}^{\mathrm{IV}}$, and $W_{\mathrm{CT}}^{\mathrm{IV}}$ are possible under different conditions (see Figure 8).



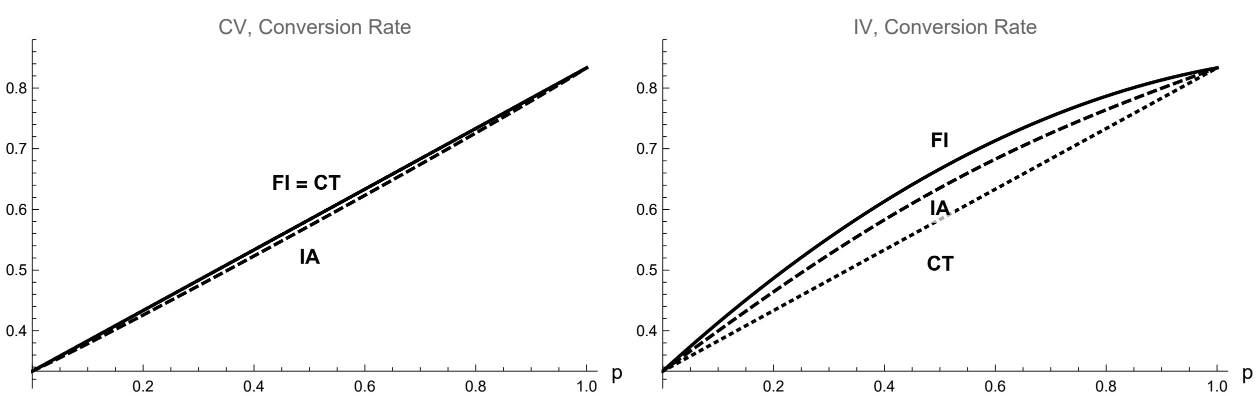
(a) Common-value case.

(b) Independent-values case.

Figure 3: Publisher's revenue under the different information settings for different values of $p \in [0,1]$, $n_1 = n_2 = 1$, $\kappa = 1/2$, and $G(x) = x$.

against an exchange advertiser with a higher valuation. Both of these bidding behaviors create an inefficient auction because an advertiser with lower valuation wins the consumer's impression, resulting in a lower conversion rate compared to the settings without information asymmetry.

Often in the literature, we see that in markets with thin competition when the publisher's revenue goes down (Figure 3(b)), conversion rate (Figure 4(b)) and the advertisers' payoffs (Figure 6) go up as we add information to the market (moving from CT to IA and then to FI). Here, we verify this for our model. However, Proposition 1 states that this is not the case when the behavioral values are correlated. In fact, both publisher revenue and conversion rate can move in the same direction, as we observe in Figures 3(a) and 4(a) in contrast to Figures 3(b) and 4(b).

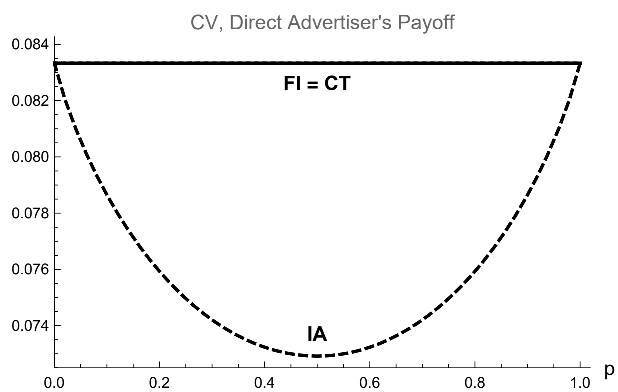


(a) Common-value case.

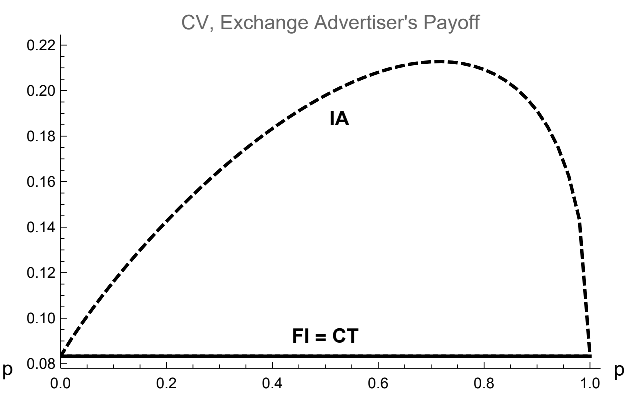
(b) Independent-values case.

Figure 4: Conversion rate under the different information settings as a function of $p \in [0,1]$, for $n_1 = n_2 = 1$, $\kappa = 1/2$, and $G(x) = x$.

Regarding advertisers' payoffs, in Figure 5 we can see that in the common-value case they change non-monotonically both in terms of the information that is available to the advertisers and in terms of p. First, in Figure 5(a), we see that the direct advertiser's payoff decreases slightly in the IA setting compared to the FI and CT settings, while in Figure 5(b) we see that the exchange advertiser's payoff increases significantly. This is expected because the exchange advertiser has a strong competitive advantage in the IA setting, while in the FI and CT settings both advertisers are similar. Second, in terms of p, we see that in the IA setting, the direct advertiser's payoff is minimum for $p = 1/2$ where the uncertainty about the common behavioral value is maximized. However, the exchange advertiser's payoff is maximized for a value $p > 1/2$, which gives the exchange advertiser a higher probability of a high valuation in addition to the advantageous uncertainty of the direct advertiser.



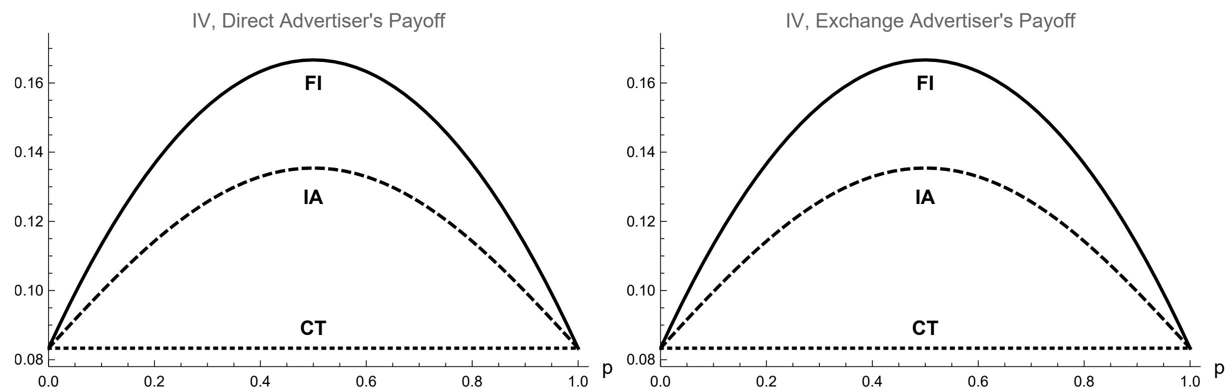
(a) Direct advertiser.



(b) Exchange advertiser.

Figure 5: Advertisers' payoffs in the common-value case under the different information settings for different values of $p \in [0, 1]$, $n_1 = n_2 = 1$, $\kappa = 1/2$, and $G(x) = x$.

In contrast to Figure 5, in the independent-values case in Figure 6 we see that both payoffs go down monotonically as we remove information from the market. Furthermore, we see that both types of advertisers have identical payoffs in all settings under IV, including the IA setting where the exchange advertiser would normally be expected to have an advantage. Although the direct advertiser has more fluctuations in their payoff in IA under different realizations of the valuations, their average payoff is the same as the exchange advertiser's one, because the advantage of extra information is not that big when the valuations are independent. This perhaps surprising result is independent of any distributional assumptions, but it is a consequence of the fact that there are only two advertisers in the simple version of the model. In Section 6 we discuss the more general case, which is more intuitive in the sense that the exchange advertiser has a higher payoff under the IA setting, but still interesting in terms of how the payoff changes as a function of p (see Figure 10).



(a) Direct advertiser. (b) Exchange advertiser.

Figure 6: Advertisers' payoffs in the independent-values case under the different information settings for different values of $p \in [0, 1]$, $n_1 = n_2 = 1$, $\kappa = 1/2$, and $G(x) = x$.

5 Generalizations

In this section, we consider a more general version of the main model, for an arbitrary behavioral distribution G (instead of uniform) and more than two advertisers. More specifically, there are $n \geq 2$ advertisers competing for the impression, a subset of $n_1 \leq n$ of them are exchange advertisers, and the remaining $n_2 = n - n_1$ are direct advertisers. For the common-value case, the results shown in Section 4.1 extend to the more general setting; this is discussed in subsection 5.1. In the independent-values case there are some interesting differences when we increase the number of advertisers, which we discuss in subsection 5.2.

5.1 Common-value case

Lemma 2 is an analog of Lemma 1 for arbitrary distributions G and more than one exchange advertisers.

Lemma 2 (Advertisers' bidding behavior)**.** *For any distribution G, any $n_1 \geq 1$, $n_2 = 1$, and $\kappa \geq 1/2$, under the common-value* IA *setting, all the exchange advertisers bid their true valuations* *while there exists* $\underline{c}(p) \in [0,1]$ *such that the direct advertiser's bidding function is*

$$\beta(c) := \begin{cases} (1-\kappa)c, & \text{if } 0 \le c < \underline{c}(p), \\ \kappa + (1-\kappa)c, & \text{if } \underline{c}(p) \le c < 1. \end{cases}$$

Moreover, $\underline{c}$ *is independent of* κ*, and it is a continuously differentiable decreasing function in* p*, with* $\underline{c}(0) = 1$*,* $\underline{c}(1/2) = n_1 \mathbb{E}[c \cdot G(c)^{n_1 - 1}]$*, and* $\underline{c}(1) = 0$*.*

Like in Lemma 1, we see that the direct advertiser sometimes underbids, for low values of c, and sometimes overbids, for high values of c. Also, as p increases, they overbid more than they underbid. Due to this non-truthful bidding, similar results to those in Section 4.1 continue to hold for $n_1 > 1$.[16] Propositions 5 and 6 generalize the results of Propositions 1 and 2 for arbitrary distributions G. Proposition 5 is shown here for any number of advertisers $n_1, n_2 \ge 1$. We further check the robustness of Proposition 6 for $n_1, n_2 > 1$ in Section 6.

Proposition 5. *For any distribution* G*, any* $n_1, n_2 \ge 1$*, and* $\kappa \in [0,1]$*, under the common-value case, we have that* $V_{\text{IA}}^{\text{CV}} \le V_{\text{FI}}^{\text{CV}} = V_{\text{CT}}^{\text{CV}}$*.*

Proposition 6. *For any distribution* G*,* $n_1 = n_2 = 1$*, and* $\kappa \ge 1/2$*, under the common-value case, we have that* $W_{\text{IA}}^{\text{CV}} \le W_{\text{FI}}^{\text{CV}} = W_{\text{CT}}^{\text{CV}}$*.*

5.2 Independent-values case

Propositions 7 and 8 generalize the results of Propositions 3 and 4. The intuition for Proposition 7 is similar to that in the simple model version (as the amount of information available to advertisers decreases, the efficiency of the auction decreases).

Proposition 7. *For any distribution* G*, any* $n_1, n_2 \ge 1$*, and* $\kappa \in [0,1]$*, under the independent-values case, we have that* $V_{\text{FI}}^{\text{IV}} \ge V_{\text{IA}}^{\text{IV}} \ge V_{\text{FI}}^{\text{IV}}$*.*

Proposition 8. *For any distribution* G*,* $n_1 = n_2 = 1$*, and* $\kappa \in [0,1]$*, under the independent-values case, we have that* $W_{\text{FI}}^{\text{IV}} \le W_{\text{IA}}^{\text{IV}} \le W_{\text{CT}}^{\text{IV}}$*.*

[16] Lemma 3 in Section 6 generalizes this result for $n_2 > 1$ as well.

In contrast to the common-value setting, under independent behavioral values, publisher revenue behaves somewhat differently in general (for $n \geq 2$) than what we showed in Propositions 4 and 8. Proposition 9 describes the general phenomenon.

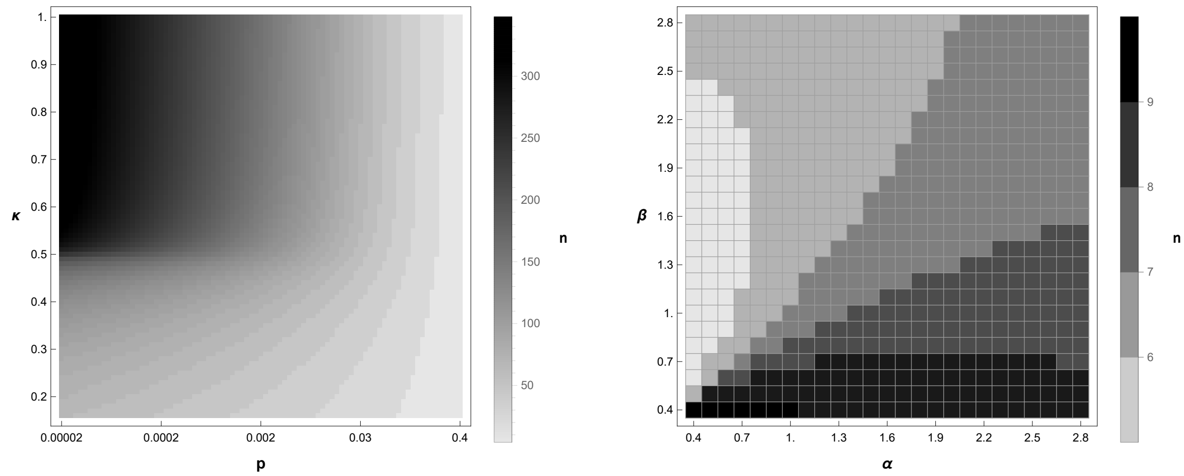
Proposition 9. *For any distribution G and $\kappa \geq 1/2$, under the independent-values case, we have $W_{\text{FI}}^{\text{IV}} \leq W_{\text{CT}}^{\text{IV}}$ for sufficiently small n, and $W_{\text{FI}}^{\text{IV}} > W_{\text{CT}}^{\text{IV}}$ for sufficiently large n. The threshold for n where the inequality is reversed depends on p, κ, and G.*

As described in Section 4.2, the market-thinning effect that occurs under the IV setting makes hiding information from advertisers beneficial for the publisher's revenue when the number of advertisers is low. However, when there is a sufficiently large number of advertisers, revealing more information increases revenue.

The threshold for the number of advertisers n where the inequality in Proposition 9 reverses depends on the parameters p and κ, and the distribution G. Figure 7 illustrates this. In Figure 7(a) we see that as p decreases and as κ increases, we need more and more advertisers to make the full-information setting give higher revenue than the contextual-targeting setting (i.e. $W_{\text{FI}}^{\text{IV}} \geq W_{\text{CT}}^{\text{IV}}$). In Figure 7(b) we see the thresholds for some examples of different Beta distributions for various parameters α and β. Generally, we observe that contextual distributions G with higher average have higher threshold. Also, if the average is low, a lower variance gives a higher threshold, but when the variance is high, a higher variance gives a higher threshold.

Given Proposition 9, a natural question to ask is how $W_{\text{IA}}^{\text{IV}}$ (the revenue under independent values with information asymmetry) compares to $W_{\text{FI}}^{\text{IV}}$ and $W_{\text{CT}}^{\text{IV}}$ under different conditions. The intuition behind Proposition 9 potentially suggests that $W_{\text{FI}}^{\text{IV}} \leq W_{\text{IA}}^{\text{IV}} \leq W_{\text{CT}}^{\text{IV}}$ for low n and $W_{\text{FI}}^{\text{IV}} \geq W_{\text{IA}}^{\text{IV}} \geq W_{\text{CT}}^{\text{IV}}$ for high n. Surprisingly, this is not always the case. In fact, as illustrated in Figure 8, all six different orderings between the revenues $W_{\text{FI}}^{\text{IV}}$, $W_{\text{IA}}^{\text{IV}}$, and $W_{\text{CT}}^{\text{IV}}$ are possible under different conditions. The information asymmetry between advertisers adds an additional element of complexity that the market-thinning effect alone is not sufficient to explain.

The intuition behind Figure 8 is as follows. In Proposition 9 we saw that a low n makes hiding information from advertisers beneficial for the publisher, due to a thinner market. For a similar reason, a low p also makes hiding information beneficial. This is because when p is low, there is a low probability that the second highest bidder at the auction will have a behavioral value $b_i = 1$,



(a) $G(x) = x$, various p and κ.

(b) $G \sim \textsc{Beta}(\alpha, \beta)$, $p = 1/20$, $\kappa = 1/2$.

Figure 7: Minimum number of advertisers n such that $W_{\text{FI}}^{\text{IV}} \geq W_{\text{CT}}^{\text{IV}}$ for various values of p, κ, and different contextual distributions G.

which means that the clearing price will most likely be of the form $(1-\kappa)c_i$ if all advertisers know their behavioral values. Thus, when p is low, the publisher prefers to hide information from as many advertisers as possible to make them bid their expected valuation $\kappa p + (1-\kappa)c_i$ instead of their actual valuation. On the contrary, when p is high, the publisher prefers to reveal the behavioral information to as many advertisers as possible so that they can bid their (likely high) actual valuation. In other words, when p is low we have that $W_{\text{FI}}^{\text{IV}} < W_{\text{IA}}^{\text{IV}} < W_{\text{CT}}^{\text{IV}}$ (Region 1) and when p is high we have that $W_{\text{FI}}^{\text{IV}} > W_{\text{IA}}^{\text{IV}} > W_{\text{CT}}^{\text{IV}}$ (Region 2). This also explains why in Regions 1, 3, and 4 it is $W_{\text{FI}}^{\text{IV}} < W_{\text{CT}}^{\text{IV}}$, while in Regions 2, 5, and 6 it is $W_{\text{FI}}^{\text{IV}} > W_{\text{CT}}^{\text{IV}}$.

To understand why the IA setting generates higher revenue than the other two information settings in Regions 3 and 6 where p is medium and κ is high, let us consider the extreme case where $\kappa = 1$. In this extreme case, the valuations of the advertisers are just b_i, without a contextual element. Under the IA setting, the exchange advertisers will bid their actual values b_i, while the direct advertisers will bid their expected valuation which is just p. When p is high, there is a high chance that there will be at least two advertisers with high b_i's, so the publisher wants the advertisers to learn their b_i's to have a high chance of getting a clearing price of 1 (Region 2). When p is low, it is less likely that there will be at least two b_i's that are high, so the publisher prefers if the advertisers bid p instead of their b_i which is more likely 0. However, having many advertisers bidding p has no additional benefit compared to just two advertisers bidding p, since

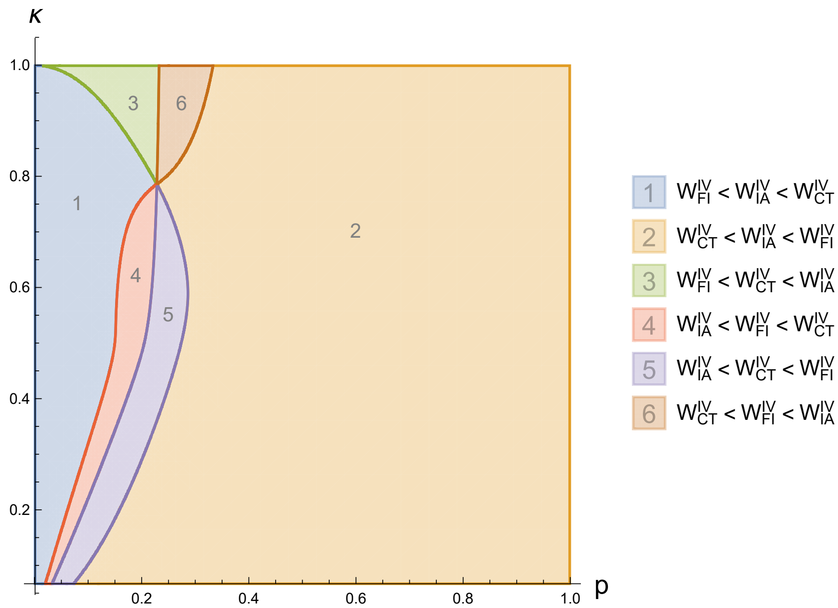


Figure 8: Publisher's revenues comparisons between different information settings under independent behavioral values, for various values of p and κ, $G(x) = x$, and $n_1 = n_2 = 2$.

the clearing price will be p in both cases. Therefore, the optimal revenue for the publisher when p is low is achieved when there is information asymmetry, where the publisher guarantees a clearing price of at least p from the direct advertisers and there is also a (small) chance of something higher from the exchange advertisers (Regions 3 and 6).

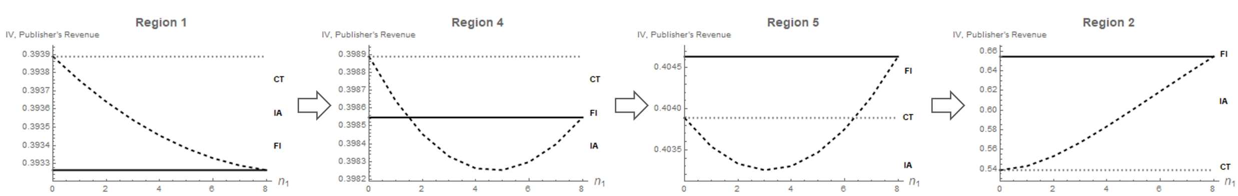


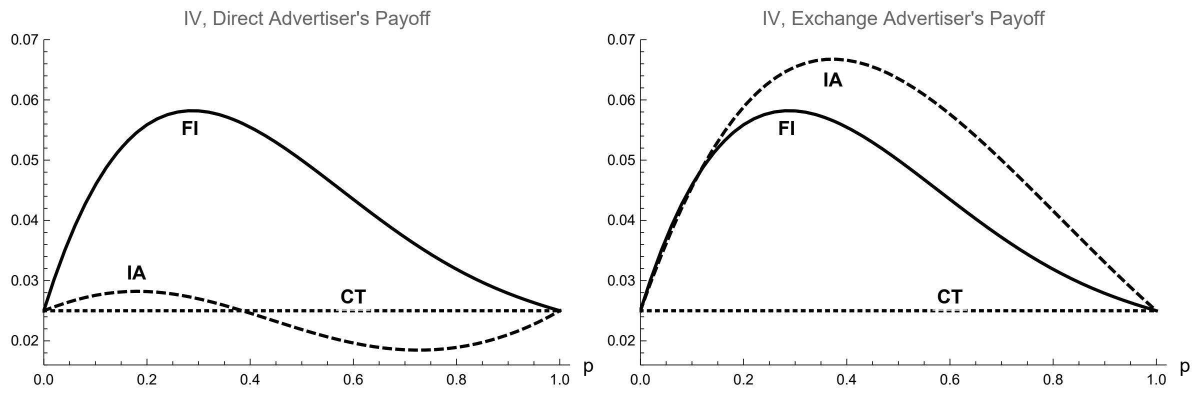
Figure 9: Publisher's revenues for the different information settings under independent behavioral values, for $n = 8$ advertisers, $n_1 \in [0, n]$, $n_2 = n - n_1$, $\kappa = 1/2$, $G(x) = x$, and $p \in \{0.01, 0.02, 0.03, 0.3\}$ (from left to right).

Finally, when κ is low, the importance of behavioral value on the advertisers' valuations is low. The contextual part of the valuations dominates in determining the winner. As a result, the benefit of information asymmetry described above, where it is good for the publisher to have both direct and exchange advertisers, is not essential anymore since the contextual values are known by both. In Regions 4 and 5, the IA setting has worse revenue than the other two settings because of a third effect.

Under the IA setting, there are two groups of advertisers, n_1 exchange advertisers and n_2 direct advertisers. If we fix the total number of advertisers $n = n_1 + n_2$, then we can think of the FI and the CT information settings as extreme versions of the IA setting. More specifically, FI is like IA with $(n_1, n_2) = (n, 0)$ and CT is like IA with $(n_1, n_2) = (0, n)$. With that view in mind, to understand how $W_{\text{FI}}^{\text{IV}}$, $W_{\text{IA}}^{\text{IV}}$, and $W_{\text{CT}}^{\text{IV}}$ compare to each other, it is useful to look at the function $W_{\text{IA}}^{\text{IV}}(n_1, n_2) = W_{\text{IA}}^{\text{IV}}(n_1, n - n_1)$ as n_1 goes from 0 to n, while everything else is fixed. In Figure 9 we can see some examples of this function (represented by the dashed line) for four different values of p, starting from a low p in the first plot on the left and increasing it towards the right (we also consider $n = 8$ advertisers to make the effect clearer). For $n_1 = 0$ the function gives the revenue under the CT setting (dotted line) and for $n_1 = n$ it gives the revenue under the FI setting (solid line). We see that for low p this function is decreasing and it gradually becomes increasing as p increases. While it transitions from decreasing to increasing, at some point, for medium values of p it becomes non-monotone (first decreasing and then increasing). This is the point where the IA setting can give lower revenue for the publisher than both the FI and the CT settings (Regions 4 and 5 in Figure 8).

The explanation for this is as follows. As n_1 increases from 0 to n, what we do is we move advertisers one by one from the group of direct advertisers to the group of exchange advertisers. The average of the bids in both groups is the same; therefore, the average bid is not affected as we move advertisers. However, what changes is the variance of the distribution of the bids. More specifically, the bids of the direct advertisers are more concentrated around the mean, while the bids of the exchange advertisers are more spread out. When we move the first few advertisers from the direct group to the exchange group, we make the bid distribution of the direct group slightly worse. However, the advertiser who determines the clearing price of the overall auction is still more likely in the direct group, as it has significantly more advertisers. Therefore, what happens is that as we start moving advertisers, we make the clearing price lower. However, after we reach a critical mass of advertisers in the exchange group, suddenly the clearing price will more likely be determined by the exchange group (i.e. there is a high chance that there will be at least two exchange advertisers with high b_i's). From that point onwards, as we make the exchange group larger, we increase the expected clearing price. This is the reason for the non-monotonicity of the function in the second and third plots of Figure 9. This transition phase is also what explains the existence of Regions 4 and 5 in Figure 8.

All three effects described above combined generate the six different regions we see in Figure 8.



(a) Direct advertiser.

(b) Exchange advertiser.

Figure 10: Advertisers' payoffs in the independent-values case under the different information settings for different values of $p \in [0,1]$, $n_1 = n_2 = 2$, $\kappa = 1/2$, and $G(x) = x$.

Advertisers' payoffs in the independent-values case. In Figure 10 we can see the payoffs of each type of advertiser for the three information settings and different values of p in $[0,1]$ when the behavioral valuations of the advertisers are independent. The two plots of Figure 10 describe the more general behavior of the payoffs when there are more than two advertisers in total (in contrast to Figure 6 which was for one advertiser of each type). There are a few interesting things to note regarding the payoffs. First, in Figure 10(a), we see that for low values of p it is $D^{\mathrm{IV}}_{\mathrm{CT}} \leq D^{\mathrm{IV}}_{\mathrm{IA}} \leq D^{\mathrm{IV}}_{\mathrm{FI}}$, while for high values of p it is $D^{\mathrm{IV}}_{\mathrm{IA}} \leq D^{\mathrm{IV}}_{\mathrm{CT}} \leq D^{\mathrm{IV}}_{\mathrm{FI}}$. In other words, when p is low, a direct advertiser prefers the asymmetric setting where exchange advertisers have more information than them, over the contextual targeting setting where all advertisers have similar information. Second, in Figure 10(b), we see that for low values of p it is $E^{\mathrm{IV}}_{\mathrm{CT}} \leq E^{\mathrm{IV}}_{\mathrm{IA}} \leq E^{\mathrm{IV}}_{\mathrm{FI}}$, while for high values of p it is $E^{\mathrm{IV}}_{\mathrm{CT}} \leq E^{\mathrm{IV}}_{\mathrm{FI}} \leq D^{\mathrm{IV}}_{\mathrm{IA}}$. In other words, when p is low, an exchange advertiser prefers the full-information setting where direct advertisers have as much information as them, over the asymmetric setting where the exchange advertiser has more information than the direct advertisers.

The explanation for these observations is the following. As an advertiser, it is often advantageous for you if other advertisers gain more information than they currently have. This is because when an advertiser does not know their actual valuation they bid their expected valuation, but when p is low it is more likely than not that their actual valuation is lower than their expected valuation.

In other words, when p is sufficiently low, you want the other advertisers to learn their actual valuations because then it is very likely that they will lower their bids.

6 More Robustness Checks

In this section, we check the robustness of Proposition 6, which is the remaining result not proven analytically for the case where $n_1, n_2 > 1$, due to the lack of a closed-form general bidding function for the direct advertisers under the common-value IA setting. We first start by establishing the existence of a pure strategy symmetric equilibrium bidding function for the general case.

Lemma 3 (Advertisers' bidding behavior)**.** *For any strictly increasing and smooth distribution G, any $n_1, n_2 \geq 1$, and $\kappa \geq 1/2$, under the common-value* IA *setting, all exchange advertisers bid their true valuations and there exists a pure strategy symmetric equilibrium bidding function β for the direct advertisers satisfying $\beta(c) \in \{(1-\kappa)c\} \cup [\kappa, \kappa + (1-\kappa)c]$ for $c \in [0,1]$.*

Lemma 3 is a generalization of Lemmas 1 and 2. Based on Lemma 3, we can numerically approximate the function β for any $n_1, n_2 \geq 1$ by solving the differential equation $\left.\frac{\partial u(\tilde{\beta};\beta,c)}{\partial \tilde{\beta}}\right|_{\tilde{\beta}=\beta(c)} = 0$, where u is defined in equation (5). In Figure 11 we can see one example of the equilibrium bidding function when there are two exchange advertisers and two direct advertisers. Like in Lemma 1, for small contextual values c, direct advertisers underbid, while for large values of c they overbid.

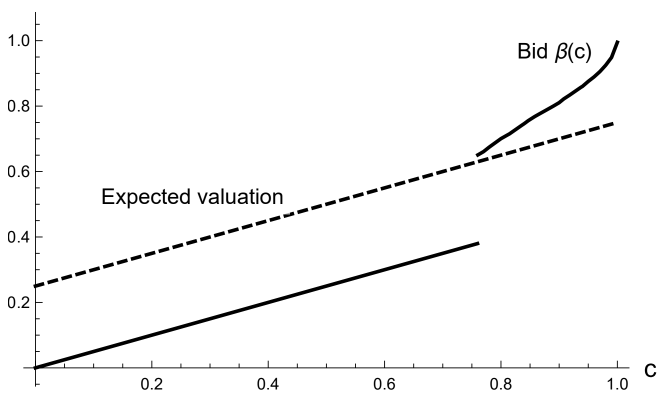
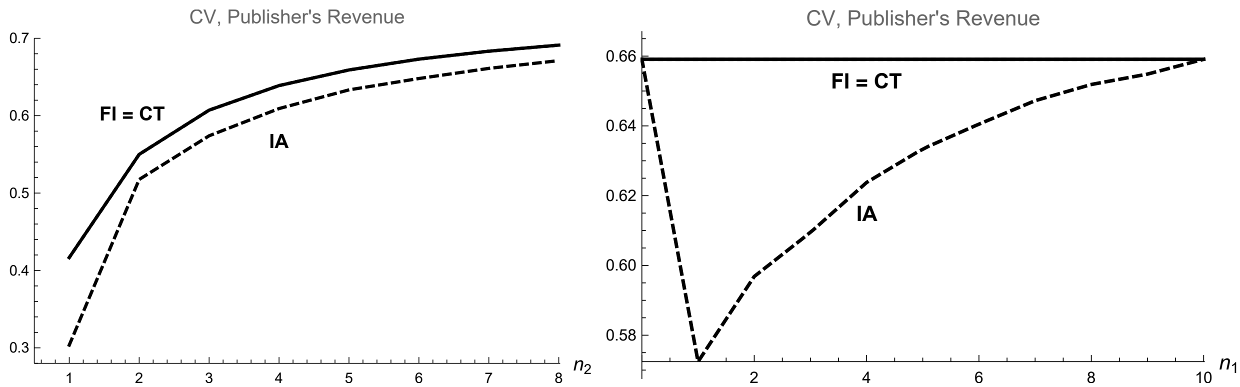


Figure 11: Bidding function of the direct advertisers (solid line) compared to their expected valuation (dashed line), for $n_1 = n_2 = 2$, $p = 1/2$, $\kappa = 1/2$, and $G(x) = x$.

Despite the lack of a closed-form bidding function, the intuition for the bidding behavior is the same as the one discussed in Section 4.1. As a result, Proposition 6 continues to hold for a



(a) $n \in \{2, 4, 6, \ldots, 16\}$, $n_1 = n_2 = n/2$.

(b) $n = 10$, $n_1 \in \{0, 1, \ldots, n\}$, $n_2 = n - n_1$.

Figure 12: Publisher's revenue under the different information settings in the common-value case for different combinations of $n_1, n_2 \geq 1$, $p = 1/2$, $\kappa = 1/2$, and $G(x) = x$.

large number of advertisers. In Figure 12 we can see a demonstration of this. In Figure 12(a) we consider different values of n, i.e. the total number of advertisers, and assuming that there is an equal number of exchange and direct advertisers, we estimate the bidding function of the direct advertisers and calculate the publisher's revenue. We see that for all cases it is $W_{\text{IA}}^{\text{CV}} \leq W_{\text{FI}}^{\text{CV}} = W_{\text{CT}}^{\text{CV}}$. In Figure 12(b) we fix the total number of advertisers n and consider all different combinations of n_1 and n_2. As before, we establish that $W_{\text{IA}}^{\text{CV}} \leq W_{\text{FI}}^{\text{CV}} = W_{\text{CT}}^{\text{CV}}$ for all cases. Different choices for the number of advertisers and the other parameters generate similar plots (see also Appendix A.2 for all the key formulas used to generate the plots).

7 Conclusion

In this paper, we study the role of information asymmetry in microtargeted online advertising. Figure 13 provides a concise summary of the main results. We find that, under certain conditions, disallowing behavioral targeting can simultaneously increase ads' conversion rates and the publisher's revenue. This new result aligns with recent empirical evidence and offers a potential explanation for what has been observed in practice.

More specifically, we demonstrate that when advertisers' behavioral valuations for a consumer are correlated, and some advertisers have access to the consumer's behavioral information while others do not, it is occasionally advantageous for the publisher, both in terms of revenue and conversion rate, to conceal all behavioral data from all advertisers (i.e., to prohibit microtargeting

Should the publisher **disable microtargeting** to maximize **conversion rate**?

	Asymmetric Information	Symmetric Information
Common Behavioral Value	**Yes** $V_{\mathrm{IA}}^{\mathrm{CV}} \leq V_{\mathrm{CT}}^{\mathrm{CV}}$	**No** $V_{\mathrm{FI}}^{\mathrm{CV}} = V_{\mathrm{CT}}^{\mathrm{CV}}$
Independent Behavioral Values	**No** $V_{\mathrm{IA}}^{\mathrm{IV}} \geq V_{\mathrm{CT}}^{\mathrm{IV}}$	**No** $V_{\mathrm{FI}}^{\mathrm{IV}} \geq V_{\mathrm{CT}}^{\mathrm{IV}}$

Should the publisher **disable microtargeting** to maximize **revenue**?

	Asymmetric Information	Symmetric Information
Common Behavioral Value	**Yes** $W_{\mathrm{IA}}^{\mathrm{CV}} \leq W_{\mathrm{CT}}^{\mathrm{CV}}$	**No** $W_{\mathrm{FI}}^{\mathrm{CV}} = W_{\mathrm{CT}}^{\mathrm{CV}}$
Independent Behavioral Values	**It depends.** (See Figure 8.)	

Figure 13: Summary of the main results.

and only permit targeting based on contextual information). This phenomenon does not occur if all advertisers share the same information or if the behavioral valuations of the advertisers are independent.

The rationale behind this result is that information asymmetry in a market where advertisers have correlated valuations can lead to inefficient bidding behavior. Some advertisers with high valuations might underbid due to concerns about overpaying, while others with low valuations may overbid for fear of losing valuable consumers. Both behaviors can result in a less efficient match between the consumer and the winning advertiser. Restricting access to behavioral information for all advertisers often leads to a less efficient market, because advertisers now have less information on which to base their bids. However, the level of inefficiency caused by the presence of asymmetric information is also significant; therefore, creating a level playing field among advertisers by eliminating microtargeting can lead to a more efficient market overall.

This paper has several important implications for key stakeholders within the online display advertising market. For publishers, our findings indicate that the information asymmetry among advertisers participating in advertising auctions may yield unexpected consequences on both revenue and ad conversion rates. Hence, a careful approach towards the type and amount of information shared with advertisers is crucial, as more information is not always better, potentially implementing measures such as disabling third-party cookies and behavioral targeting for a more efficient ad market. For advertisers, while more information about impressions they bid for is generally advantageous, less information for competitors may not always be beneficial. The bidding behavior of competitors could negatively impact them given the dynamics of the market, underlining the need for a more fair and balanced market. Moreover, their bidding strategies may need to be flexible, potentially requiring underbidding or overbidding depending on the specific information available to them, for optimal results. Regarding regulators, both the NPO (Edelman, 2020) and the New York Times (Davies, 2019) examples from Section 1 were responses from publishers to the General Data Protection Regulation (GDPR) in the European Union. It is interesting to note that such data privacy protection laws, initially conceived to protect consumers, can also inadvertently benefit publishers and advertisers. Therefore, careful design of these regulations can result in a win-win scenario for all parties involved. Lastly, from a consumer standpoint, higher conversion rates usually imply more satisfactory ad content; therefore, disabling third-party tracking on a publisher's website can offer additional benefits beyond enhancing consumer privacy. It can also improve the relevance of the ads, supporting the growing trend of websites offering users the option to opt-out from tracking.

An interesting direction for future research is to examine the impact of microtargeting on consumer behavior. Consumers might prefer publishers who refrain from disclosing behavioral information to advertisers. This preference could influence publishers' decisions to enable or disable behavioral targeting, thereby affecting both publishers' revenue and conversion rates. If this is the case, it could serve as an additional mechanism that explains the increase in conversion rates when disallowing behavioral targeting.

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A Appendix

A.1 Analysis and Proofs

Proof of Lemma 1

The direct advertiser's expected utility when their contextual value is c and they bid β is:

$$u(\beta,c) := p(1-\kappa)\int_0^{\max\{\frac{\beta-\kappa}{1-\kappa},0\}}(c-c')d(G(c')^{n_1}) + (1-p)(1-\kappa)\int_0^{\min\{\frac{\beta}{1-\kappa},1\}}(c-c')d(G(c')^{n_1}).$$

Suppose that $\kappa < 1-\kappa$. First, let us consider the case when $0 \le \beta < \kappa$, we have

$$u(\beta,c) = (1-p)(1-\kappa)\int_0^{\frac{\beta}{1-\kappa}}(c-c')d(G(c')^{n_1}),$$

which means,

$$\frac{\partial u}{\partial \beta} = n_1(1-p)\left(c - \frac{\beta}{1-\kappa}\right)G\left(\frac{\beta}{1-\kappa}\right)^{n_1-1}G'\left(\frac{\beta}{1-\kappa}\right) = 0 \quad \Longrightarrow \quad \beta = (1-\kappa)c.$$

For, the case when $\kappa < \beta \le 1-\kappa$, we have

$$u(\beta,c) = p(1-\kappa)\int_0^{\frac{\beta-\kappa}{1-\kappa}}(c-c')d(G(c')^{n_1}) + (1-p)(1-\kappa)\int_0^{\frac{\beta}{1-\kappa}}(c-c')d(G(c')^{n_1}),$$

which means,

$$\frac{\partial u}{\partial \beta} = n_1 p\left(c - \frac{\beta-\kappa}{1-\kappa}\right)G\left(\frac{\beta-\kappa}{1-\kappa}\right)^{n_1-1}G'\left(\frac{\beta-\kappa}{1-\kappa}\right) + n_1(1-p)\left(1 - \frac{\beta}{1-\kappa}\right)G\left(\frac{\beta}{1-\kappa}\right)^{n_1-1}G'\left(\frac{\beta}{1-\kappa}\right).$$

When $n_1 = 1$ and $G(x) = x$, $\frac{\partial u}{\partial \beta} = 0$ implies that $\beta = \kappa p + (1-\kappa)c$. For the case when $1-\kappa < \beta \le 1$, we have

$$u(\beta,c) = p(1-\kappa)\int_0^{\frac{\beta-\kappa}{1-\kappa}}(c-c')d(G(c')^{n_1}) + (1-p)(1-\kappa)\int_0^{1}(c-c')d(G(c')^{n_1}),$$

which means,

$$\frac{\partial u}{\partial \beta} = n_1 p\left(c - \frac{\beta-\kappa}{1-\kappa}\right)G\left(\frac{\beta-\kappa}{1-\kappa}\right)^{n_1-1}G'\left(\frac{\beta-\kappa}{1-\kappa}\right) = 0 \quad \Longrightarrow \quad \beta = \kappa + (1-\kappa)c.$$

The global maximum of u occurs either at $\beta = (1-\kappa)c, \kappa p + (1-\kappa)c, \kappa + (1-\kappa)c$, or at one of the singular points $\beta = \kappa, 1-\kappa$. Let

$$u_1(c) := u(\beta = (1-\kappa)c, c) = \begin{cases} \frac{1}{2}(1-p)(1-\kappa)c^2, & c \leq \frac{\kappa}{1-\kappa}, \\ \frac{1}{2}(1-\kappa)c^2 - \frac{p}{2}\left(\frac{\kappa^2}{1-\kappa}\right), & \frac{\kappa}{1-\kappa} < c \leq 1, \end{cases}$$

$$u_2(c) := u(\beta = \kappa p + (1-\kappa)c, c) = \begin{cases} \frac{1}{2}(1-p)(1-\kappa)c^2 - \frac{1}{2}(1-p)p^2\left(\frac{\kappa^2}{1-\kappa}\right), & c \leq \frac{(1-p)\kappa}{1-\kappa}, \\ \frac{1}{2}(1-\kappa)c^2 - \frac{1}{2}(1-p)p\left(\frac{\kappa^2}{1-\kappa}\right), & \frac{(1-p)\kappa}{1-\kappa} < c \leq 1 - \frac{p\kappa}{1-\kappa}, \\ \frac{1}{2}p(1-\kappa)c^2 - \frac{1}{2}p(1-p)^2\left(\frac{\kappa^2}{1-\kappa}\right) & \\ \qquad +(1-p)(1-\kappa)\left(c - \frac{1}{2}\right), & 1 - \frac{p\kappa}{1-\kappa} < c \leq 1, \end{cases}$$

$$u_3(c) := u(\beta = \kappa + (1-\kappa)c, c) + \begin{cases} \frac{1}{2}(1-\kappa)c^2 - \frac{1-p}{2}\left(\frac{\kappa^2}{1-\kappa}\right), & c \leq 1 - \frac{\kappa}{1-\kappa}, \\ \frac{1}{2}p(1-\kappa)c^2 + (1-p)(1-\kappa)\left(c - \frac{1}{2}\right), & 1 - \frac{\kappa}{1-\kappa} < c \leq 1, \end{cases}$$

$$u_4(c) := u(\beta = \kappa, c) = (1-p)(1-\kappa)\int_0^{\frac{\kappa}{1-\kappa}} (c - c')d(G(c')^{n_1}) = (1-p)\kappa c - \frac{1}{2}(1-p)\left(\frac{\kappa^2}{1-\kappa}\right),$$

$$\begin{aligned} u_5(c) := u(\beta = 1-\kappa, c) =& p(1-\kappa)\int_0^{\frac{1-2\kappa}{1-\kappa}} (c - c')d(G(c')^{n_1}) + (1-p)(1-\kappa)\int_0^1 (c - c')d(G(c')^{n_1}) \\ =& p(1-2\kappa)c - \frac{1}{2}p\left(\frac{(1-2\kappa)^2}{1-\kappa}\right) + (1-p)(1-\kappa)\left(c - \frac{1}{2}\right). \end{aligned}$$

Clearly, $u_4(c) \leq u_1(c)$ (in fact u_4 is tangent to $\frac{1}{2}(1-p)(1-\kappa)c^2$ at $c = \frac{\kappa}{1-\kappa}$) and $u_5(c) \leq u_3(c)$ (in fact u_5 is tangent to $\frac{1}{2}p(1-\kappa)c^2 + (1-p)(1-\kappa)\left(c - \frac{1}{2}\right)$ at $c = 1 - \frac{\kappa}{1-\kappa}$), so we can ignore u_4 and u_5. Then $\beta = (1-\kappa)c$ when $u_1(c) > u_2(c), u_3(c)$, $\beta = \kappa p + (1-\kappa)c$ when $u_2(c) > u_1(c), u_3(c)$, and $\beta = \kappa + (1-\kappa)c$ when $u_3(c) > u_1(c), u_2(c)$, and we break ties arbitrarily. We note that u_1, u_2, u_3 are all continuous in c and that $\frac{du_1}{dc} \leq \frac{du_2}{dc} \leq \frac{du_3}{dc}$, therefore u_1 can only be overtaken by u_2, u_3 and u_2 can only be overtaken by u_3, and u_3 cannot be overtaken. So, for $\kappa < 1-\kappa$, there must exist $\underline{c}$ and $\bar{c}$ such that $\beta(c) = (1-\kappa)c$ if $c < \underline{c}$, $\beta(c) = \kappa p + (1-\kappa)c$ if $\underline{c} < c < \bar{c}$, and $\beta(c) = \kappa + (1-\kappa)c$.

Let us now find c_{12}, the point where u_2 overtakes u_1, suppose that $\frac{(1-p)\kappa}{1-\kappa} \le c_{12} \le \frac{\kappa}{1-\kappa}$:

$$\frac{1}{2}(1-p)(1-\kappa)c_{12}^2 = \frac{1}{2}(1-\kappa)c_{12}^2 - \frac{1}{2}(1-p)p\left(\frac{\kappa^2}{1-\kappa}\right) \quad \Longrightarrow \quad c_{12} = \frac{\sqrt{1-p}\kappa}{1-\kappa} \in \left[\frac{(1-p)\kappa}{1-\kappa}, \frac{\kappa}{1-\kappa}\right].$$

We do not need to further check other intervals due to the uniqueness of the intersection point. Similarly, we find the location of the point c_{23} where u_3 overtakes u_2: $1 - \frac{\kappa}{1-\kappa} \le c_{23} \le 1 - \frac{p\kappa}{1-\kappa}$:

$$\frac{1}{2}p(1-\kappa)c_{23}^2 + (1-p)(1-\kappa)\left(c_{23} - \frac{1}{2}\right) = \frac{1}{2}(1-\kappa)c_{23}^2 - \frac{1}{2}(1-p)p\left(\frac{\kappa^2}{1-\kappa}\right).$$

This is a quadratic equation in c_{23} that has roots: $1 \pm \frac{\sqrt{p}\kappa}{1-\kappa}$. We take the negative root $c_{23} = 1 - \frac{\sqrt{p}\kappa}{1-\kappa} \in \left[1 - \frac{\kappa}{1-\kappa}, 1 - \frac{p\kappa}{1-\kappa}\right]$. Finally, we consider the point c_{13} where u_3 overtakes u_1, suppose that $1 - \frac{\kappa}{1-\kappa} < c_{13} < \frac{\kappa}{1-\kappa}$:

$$\frac{1}{2}(1-p)(1-\kappa)c_{13}^2 = \frac{1}{2}p(1-\kappa)c_{13}^2 + (1-p)(1-\kappa)\left(c_{13} - \frac{1}{2}\right).$$

This is a quadratic equation in c_{13} with two roots: $\frac{\sqrt{1-p}}{\sqrt{1-p}\pm\sqrt{p}}$. Since $c_{13} \in [0,1]$, we take the positive root: $c_{13} = \frac{\sqrt{1-p}}{\sqrt{1-p}+\sqrt{p}}$. Finally, we take $\underline{c} := \min\{c_{12}, c_{13}\}$, and $\bar{c} := \max\{c_{13}, c_{23}\}$, this also ensures that c_{13} is relevant only if $1 - \frac{\kappa}{1-\kappa} < c_{23} < c_{13} < c_{12} < \frac{\kappa}{1-\kappa}$.

Now, suppose that $\kappa \ge 1 - \kappa$. Let us consider the case when $0 \le \beta \le 1 - \kappa$, we have

$$u(\beta, c) = (1-p)(1-\kappa)\int_0^{\frac{\beta}{1-\kappa}} (c - c')d(G(c')^{n_1}),$$

as before, $\frac{\partial u}{\partial c} = 0$ implies $\beta = (1-\kappa)c$. For $1 - \kappa \le \beta \le \kappa$, we find that

$$u(\beta, c) = (1-p)(1-\kappa)\int_0^1 (c - c')d(G(c')^{n_1}),$$

which is a constant in β. For $\kappa < \beta \le 1$, we have

$$u(\beta, c) = p(1-\kappa)\int_0^{\frac{\beta-\kappa}{1-\kappa}} (c - c')d(G(c')^{n_1}) + (1-p)(1-\kappa)\int_0^1 (c - c')d(G(c')^{n_1}),$$

which means $\frac{\partial u}{\partial \beta} = 0$ implies $\beta = \kappa + (1-\kappa)c$. This time we let

$$u_1(c) := u(\beta = (1-\kappa)c, c) = \frac{1}{2}(1-p)(1-\kappa)c^2,$$

$$u_2(c) := u(\beta = \kappa + (1-\kappa)c, c) = \frac{1}{2}p(1-\kappa)c^2 + (1-p)(1-\kappa)\left(c - \frac{1}{2}\right).$$

And, as before, $u_3(c) := u(\beta = \kappa, c)$, $u_4(c) := u(\beta = 1-\kappa, c)$, which we can check that they satisfy $u_3(c) \leq u_1(c)$ and $u_4(c) \leq u_2(c)$, so we can ignore them. Since $\frac{du_1}{dc} = (1-p)(1-\kappa) \leq \frac{du_2}{dc} = (1-\kappa)c$, we conclude that u_2 can only overtake u_1 and cannot be overtaken. Hence, there exists $\underline{c} = \bar{c}$ such that $\beta(c) = (1-\kappa)c$ if $c < \underline{c}$ and $\beta(c) = \kappa + (1-\kappa)c$ if $c > \bar{c}$. Further inspection reveals that $\underline{c} = \bar{c} = c_{13} = \frac{\sqrt{1-p}}{\sqrt{1-p}+\sqrt{p}}$ as previously found. This completes the proof. ■

Proofs of Propositions 1, 2, 3, and 4

Propositions 1, 2, 3, and 4 are special cases of Propositions 5, 6, 7, and 8. We present the proofs of the more general statements below. ■

Proof of Lemma 2

When $\kappa \geq 1/2$, we have $\kappa \geq 1-\kappa$, and we only need to consider two cases: $0 \leq \beta \leq 1-\kappa$ where the direct advertiser expected utility is:

$$u(\beta, c) = (1-p)(1-\kappa)\int_0^{\frac{\beta}{1-\kappa}} (c - c')d(G(c')^{n_1})$$

and $\kappa \leq \beta \leq 1$ where the direct advertiser expected utility is:

$$u(\beta, c) = p(1-\kappa)\int_0^{\frac{\beta-\kappa}{1-\kappa}} (c - c')d(G(c')^{n_1}) + (1-p)(1-\kappa)\int_0^1 (c - c')d(G(c')^{n_1}).$$

We do not need to consider the $1-\kappa < \beta < \kappa$ case since

$$u(\beta, c) = (1-p)(1-\kappa)\int_0^1 (c - c')d(G(c')^{n_1})$$

is constant in β over that domain. It follows that if $0 \le \beta \le 1-\kappa$, then

$$\frac{\partial u}{\partial \beta} = n_1(1-p)\left(c - \frac{\beta}{1-\kappa}\right) G\left(\frac{\beta}{1-\kappa}\right)^{n_1-1} G'\left(\frac{\beta}{1-\kappa}\right) = 0 \qquad \Longrightarrow \qquad \beta = (1-\kappa)c.$$

Similarly, if $\kappa \le \beta \le 1$, then

$$\frac{\partial u}{\partial \beta} = n_1 p\left(c - \frac{\beta-\kappa}{1-\kappa}\right) G\left(\frac{\beta-\kappa}{1-\kappa}\right)^{n_1-1} G'\left(\frac{\beta-\kappa}{1-\kappa}\right) = 0 \qquad \Longrightarrow \qquad \beta = \kappa + (1-\kappa)c.$$

For any fixed c, the global maximum of u occurs either at $\beta = (1-\kappa)c, \kappa + (1-\kappa)c$ or at one of the singular points $\beta = \kappa, 1-\kappa$. Let

$$u_1(c) := u(\beta = (1-\kappa)c, c) = (1-p)(1-\kappa)\int_0^c (c-c')d(G(c')^{n_1}),$$

$$u_2(c) := u(\beta = \kappa + (1-\kappa)c, c) = p(1-\kappa)\int_0^c (c-c')d(G(c')^{n_1}) + (1-p)(1-\kappa)\int_0^1 (c-c')d(G(c')^{n_1}),$$

$$u_3(c) := u(\beta = \kappa, c) = (1-p)(1-\kappa)\int_0^{\frac{\kappa}{1-\kappa}} (c-c')d(G(c')^{n_1}),$$

$$u_4(c) := u(\beta = 1-\kappa, c) = p(1-\kappa)\int_0^{1-\frac{\kappa}{1-\kappa}} (c-c')d(G(c')^{n_1}) + (1-p)(1-\kappa)\int_0^1 (c-c')d(G(c')^{n_1}).$$

Since $u_3(c)$ is the tangent line to $u_1(c)$ at $c = \frac{\kappa}{1-\kappa}$ and $u_4(c)$ is the tangent line to $u_2(c)$ at $c = 1 - \frac{\kappa}{1-\kappa}$, and both u_1, u_2 are convex, we have $u_3 \le u_1$ and $u_4 \le u_2$, so we can ignore u_3, u_4. Next, we note that

$$\frac{du_1}{dc} = (1-p)(1-\kappa)G(c)^{n_1} < p(1-\kappa)G(c)^{n_1} + (1-p)(1-\kappa) = \frac{du_2}{dc}$$

for all $c \in [0,1]$. We conclude that u_1 can only be overtaken by u_2. Note also that $u_1(0) = 0 > u_2(0) = -(1-p)(1-\kappa)\int_0^1 c' d(G(c')^{n_1})$ and $u_2(1) = p(1-\kappa)\int_0^c (1-c')d(G(c')^{n_1}) + u_1(1) > u_1(1)$, so the intersection point $\underline{c}(p) \in [0,1]$ exists and is unique. For a given distribution G, we can find $\underline{c}$ from the relation $u_1(\underline{c}) = u_2(\underline{c})$. Equivalently, the relation for $\underline{c}$ may be written as

$$\int_0^1 (\underline{c} - c')d(G(c')^{n_1}) = \frac{1-2p}{1-p}\int_0^{\underline{c}} (\underline{c} - c')d(G(c')^{n_1}). \tag{2}$$

Clearly, $1-\kappa$ cancels out and $\underline{c}$ is independent of κ. Furthermore, $u_1 - u_2$ is continuously differentiable in p and in $\underline{c}$ with nonvanishing derivative, and hence $\underline{c}$ is continuously differentiable in p by the Implicit Function Theorem. Differentiating $u_1 - u_2 = 0$ with respect to p, we get:

$$-\frac{1}{1-p}\int_0^{\underline{c}}(\underline{c}-c')d(G(c')^{n_1}) = \left[(1-p)(1-G(\underline{c})^{n_1}) + pG(\underline{c})^{n_1}\right]\frac{d\underline{c}}{dp}. \tag{3}$$

The factor in the square bracket above is positive and also $\int_0^{\underline{c}}(\underline{c}-c')d(G(c')^{n_1}) > 0$, hence $\frac{d\underline{c}}{dp} < 0$.

From (2) we can see that $p = 0$ implies $\int_{\underline{c}}^1(\underline{c}-c')d(G(c')^{n_1}) = 0$, which holds exactly if $\underline{c}(0) = 1$ as the integral is < 0 for all $\underline{c} < 1$. Similarly, we can see that the LHS of (2) is bounded in $[-1,1]$, while the RHS approaches $-\infty$ as $p \to 1^-$, unless $\underline{c} \to 0$, which must be the case. Hence $\underline{c}(1) = 0$. Lastly, the RHS of (2) vanishes when $p = 1/2$, therefore, we are left with $\int_0^1(\underline{c}-c')d(G(c')^{n_1}) = 0$ or $\underline{c} = \int_0^1 c'd(G(c')^{n_1}) = \mathbb{E}[n_1 c G(c)^{n_1-1}]$, as claimed. ∎

Proof of Proposition 5

The statement of the proposition holds in a more general setting, which we prove in Lemma 4.

Lemma 4. *Consider any auction mechanism M such that, whenever bidders are symmetric and independent, in equilibrium M allocates the impression to the highest-value bidder. Then in our model under the common-value case, for any distribution G, any $n_1, n_2 \geq 1$, $\kappa \in [0,1]$, and with selling mechanism M (instead of second-price auction), we have $V_{\mathrm{IA}}^{\mathrm{CV}} \leq V_{\mathrm{FI}}^{\mathrm{CV}} = V_{\mathrm{CT}}^{\mathrm{CV}}$.*

Proof. Under both the full-information and the contextual-targeting settings, when the behavioral value b is common among all the bidders, we have that the bidders are symmetric with their valuations determined by the independently drawn contextual values c_i. Therefore, the impression is allocated to the bidder with the highest c_i under M. Under full information, the valuation of any bidder i is $v_i = v(b_i, c_i) := \kappa b_i + (1-\kappa)c_i$ where $b_i = 1$ with probability p and $b_i = 0$ with probability $1-p$. Under the contextual-targeting setting, the valuation of any bidder i is $v_i = \mathbb{E}[v(b_i, c_i)|c_i] = \kappa p + (1-\kappa)c_i$. It follows that the expected conversion rate is

$$V_{\mathrm{FI}}^{\mathrm{CV}} = \mathbb{E}_b\left[(n_1+n_2)\int_0^1 v(b,c)G(c)^{n_1+n_2-1}G'(c)dc\right]$$
$$= (n_1+n_2)\int_0^1 \mathbb{E}\left[v(b,c)|c\right]G(c)^{n_1+n_2-1}G'(c)dc = V_{\mathrm{CT}}^{\mathrm{CV}},$$

where in the second equality we applied Fubini's Theorem. Since the mechanism M under full information ensures that the bidder with the highest valuation will win, it must be the case that $V_{\text{FI}}^{\text{CV}}$ is the highest possible conversion rate under any information setting. In particular, $V_{\text{IA}}^{\text{CV}} \leq V_{\text{FI}}^{\text{CV}} = V_{\text{CT}}^{\text{CV}}$. $\square$

Proof of Proposition 6

The fact that $W_{\text{CT}}^{\text{CV}} = W_{\text{FI}}^{\text{CV}}$ is general and can be seen by directly comparing their expressions. For $n_1 = n_2 = 1$ and $\kappa \geq 1/2$ we may simplify (6) to:

$$\begin{aligned}
W_{\text{IA}}^{\text{CV}} =& 2p \int_{\underline{c}}^{1} (\kappa + (1-\kappa)c)(1-G(c))G'(c)dc + 2(1-p)\int_{0}^{\underline{c}} (1-\kappa)c(1-G(c))G'(c)dc \\
&+ p\int_{0}^{\underline{c}} (1-\kappa)cG'(c)dc + p\int_{0}^{\underline{c}} (\kappa + (1-\kappa)c)(1-G(\underline{c}))G'(c)dc + (1-p)\int_{\underline{c}}^{1}(1-\kappa)c(1-G(\underline{c}))G'(c)dc \\
=& W_{\text{FI}}^{\text{CV}} + p\left(\int_{0}^{\underline{c}}(\kappa + (1-\kappa)c)G(c)G'(c)dc - \int_{0}^{\underline{c}}(\kappa + (1-\kappa)c)(G(\underline{c}) - G(c))G'(c)dc - \kappa G(\underline{c})\right) \\
&+ (1-p)\left(\int_{\underline{c}}^{1}(1-\kappa)c(G(c) - G(\underline{c}))G'(c)dc - \int_{\underline{c}}^{1}(1-\kappa)c(1-G(c))G'(c)dc\right) \\
=:& W_{\text{FI}}^{\text{CV}} + W_{\Delta},
\end{aligned}$$

where W_{Δ} is defined to be the sum of the first and the second bracket. Let us show that $W_{\Delta} \leq 0$ for all $p \in [0,1]$. First, we note that (2) can be written equivalently as

$$pn_1 \int_{0}^{\underline{c}} cG(c)^{n_1 - 1}G'(c)dc + (1-p)n_1\int_{\underline{c}}^{1} cG(c)^{n_1-1}G'(c)dc = \underline{c}pG(\underline{c})^{n_1} + \underline{c}(1-p)(1-G(\underline{c})^{n_1}). \quad (4)$$

Then, we have

$$\begin{aligned}
W_{\Delta} =& \frac{1}{2}\kappa pG(\underline{c})^2 + (1-\kappa)p\int_{0}^{\underline{c}} cG(c)G'(c)dc \\
&- \kappa pG(\underline{c})^2 - (1-\kappa)pG(\underline{c})\int_{0}^{\underline{c}} cG'(c)dc + \frac{1}{2}\kappa pG(\underline{c})^2 + (1-\kappa)p\int_{0}^{\underline{c}} cG(c)G'(c)dc - \kappa pG(\underline{c}) \\
&+ (1-\kappa)(1-p)\int_{\underline{c}}^{1} cG(c)G'(c)dc - (1-\kappa)(1-p)G(\underline{c})\int_{\underline{c}}^{1} cG'(c)dc - (1-\kappa)(1-p)\int_{\underline{c}}^{1} c(1-G(c))G'(c)dc \\
=& 2(1-\kappa)p\int_{0}^{\underline{c}} cG(c)G'(c)dc + (1-\kappa)(1-p)\int_{\underline{c}}^{1} cG(c)G'(c)dc \\
&- (1-\kappa)G(\underline{c})p\int_{0}^{\underline{c}} cG'(c)dc - (1-\kappa)G(\underline{c})(1-p)\int_{\underline{c}}^{1} cG'(c)dc \\
&- (1-\kappa)(1-p)\int_{\underline{c}}^{1} c(1-G(c))G'(c)dc - \kappa pG(\underline{c}) \\
\leq& (1-\kappa)p\underline{c}G(\underline{c})^2 + (1-\kappa)(1-p)\int_{\underline{c}}^{1} cG(c)G'(c)dc \\
&- (1-\kappa)G(\underline{c})\left(\underline{c}pG(\underline{c}) + \underline{c}(1-p)(1-G(\underline{c}))\right) - (1-\kappa)(1-p)\int_{\underline{c}}^{1} c(1-G(c))G'(c)dc
\end{aligned}$$

$$+(1-\kappa)(1-p)\underline{c}(1-G(\underline{c}))-(1-\kappa)p\int_0^{\underline{c}} cG'(c)dc-(1-\kappa)(1-p)\int_{\underline{c}}^1 cG'(c)dc.$$

The last inequality can be explained as follows. We rewrite the first line using the inequalities: $2(1-\kappa)p\int_0^{\underline{c}} cG(c)G'(c)dc \le (1-\kappa)p\underline{c}\int_0^{\underline{c}} d(G(c)^2) = (1-\kappa)p\underline{c}G(\underline{c})^2$. The second line follows from (4) with $n_1 = 1$. Lastly, we rewrite $\kappa pG(\underline{c})$ using (4) and the fact that $1-\kappa \le \kappa$, $\underline{c} \le 1$:

$$\kappa pG(\underline{c}) \ge (1-\kappa)p\underline{c}G(\underline{c}) = (1-\kappa)p\int_0^{\underline{c}} cG'(c)dc+(1-\kappa)(1-p)\int_{\underline{c}}^1 cG'(c)d-(1-\kappa)\underline{c}(1-p)(1-G(\underline{c})).$$

Back to the main calculation, after some cancellations, the last inequality becomes:

$$\begin{aligned}
W_\Delta \le& (1-\kappa)(1-p)\underline{c}(1-G(\underline{c}))^2-2(1-\kappa)(1-p)\int_{\underline{c}}^1 c(1-G(c))G'(c)dc-(1-\kappa)p\int_0^{\underline{c}} cG'(c)dc \\
\le& (1-\kappa)(1-p)\underline{c}(1-G(\underline{c}))^2-2(1-\kappa)(1-p)\underline{c}\int_{\underline{c}}^1 (1-G(c))G'(c)dc-(1-\kappa)p\int_0^{\underline{c}} cG'(c)dc \\
=& -(1-\kappa)p\int_0^{\underline{c}} cG'(c)dc \le 0.
\end{aligned}$$

Therefore, we have that $W_\Delta \le 0$ as needed. In fact, we can see from $W_\Delta \le -(1-\kappa)p\int_0^{\underline{c}} cG'(c)dc$, that the equality holds exactly when $p = 0, 1$, i.e. $W^{\mathrm{CV}}_{\mathrm{IA}}|_{p=0,1} = W^{\mathrm{CV}}_{\mathrm{FI}}|_{p=0,1} = W^{\mathrm{CV}}_{\mathrm{CT}}|_{p=0,1}$. ■

Proof of Proposition 7

Let's consider $N := n_1 + n_2$ advertisers which are divided into two disjoint subsets $A = \{a^A_1, \cdots, a^A_{n_1}\}$ and $B = \{a^B_1, \cdots, a^B_{n_2}\}$, $A \coprod B = \{1, 2, \cdots, n_1 + n_2\}$. Set A contains exchange advertisers with full information and hence bid their true valuation. Set B contains direct advertisers with only the contextual value, and hence bid the expected value $\kappa p + (1-\kappa)c$. Let's consider an instance of an auction where the contextual values in set B are given by $c_1 > \cdots > c_{n_2}$, whereas the highest bid in A is given by the bidder a^* with valuation $\kappa b^* + (1-\kappa)c^*$. Independently, we also draw $b_1, \cdots, b_{n_2}$ behavioral values for the direct advertisers in B.

First, we consider the case where a^B_1 from B is the winner: $\kappa p + (1-\kappa)c_1 > \kappa b^* + (1-\kappa)c^*$. Suppose that we moved an advertiser $a^B_i \ne a^B_1$ from set B to set A, keeping all the contextual values fixed. After the move, either a^B_i becomes the winner or nothing changes. Suppose a^B_i becomes the winner, this means $b_i = 1$, $\kappa + (1-\kappa)c_i > \kappa b^* + (1-\kappa)c^*$ and

$$\kappa + (1-\kappa)c_i > \kappa p + (1-\kappa)c_1 \qquad \Longrightarrow \qquad c_1 - c_i < \frac{\kappa(1-p)}{1-\kappa}.$$

With probability p we have $b_1 = 1$, and in this case, the change in the winner's valuation Δv_w is given by

$$\mathbb{E}[\Delta v_w | b_1 = 1] = (\kappa + (1-\kappa)c_i) - (\kappa + (1-\kappa)c_1) = -(1-\kappa)(c_1 - c_i) > -\kappa(1-p).$$

With probability $1-p$ we have $b_1 = 0$, and in this case, the change in the winner's valuation is given by

$$\mathbb{E}[\Delta v_w | b_1 = 0] = (\kappa + (1-\kappa)c_i) - (1-\kappa)c_1 = \kappa - (1-\kappa)(c_1 - c_i) > \kappa - \kappa(1-p) = \kappa p.$$

Therefore, the expected change of the winner's valuation is

$$\mathbb{E}[\Delta v_w] = \mathbb{E}[\Delta v_w | b_1 = 1]\mathbb{P}[b_1 = 1] + \mathbb{E}[\Delta v_w | b_1 = 0]\mathbb{P}[b_1 = 0] > -p \cdot \kappa(1-p) + (1-p) \cdot \kappa p = 0.$$

Suppose that we moved an advertiser a_1^B from set B to A, keeping all the drawn contextual values fixed. After the move, either a_1^B remains the winner, hence nothing changes, or it is not. If a_1^B is no longer a winner, then either a^* is the winner, in that case, we have an increase in the winner's valuation since $\kappa b^* + (1-\kappa)c^* > \kappa b_1 + (1-\kappa)c_1$. Otherwise, a_2^B is now the winner, so we must have $b_1 = 0$ and

$$\kappa p + (1-\kappa)c_2 > (1-\kappa)c_1 \qquad \Longrightarrow \qquad c_1 - c_2 < \frac{\kappa p}{1-\kappa}.$$

With probability p we have $b_2 = 1$, and in this case, the change in the winner's valuation is given by

$$\mathbb{E}[\Delta v_w | b_2 = 1] = (\kappa + (1-\kappa)c_2) - (1-\kappa)c_1 = \kappa - (1-\kappa)(c_1 - c_2) > \kappa - \kappa p = \kappa(1-p).$$

With probability $1-p$ we have $b_2 = 0$, and in this case, the change in the winner's valuation is given by

$$\mathbb{E}[\Delta v_w | b_2 = 0] = (1-\kappa)c_2 - (1-\kappa)c_1 = -(1-\kappa)(c_1 - c_2) > -\kappa p.$$

Therefore, the expected change of the winner's valuation is

$$\mathbb{E}[\Delta v_w] = \mathbb{E}[\Delta v_w | b_1 = 1]\mathbb{P}[b_1 = 1] + \mathbb{E}[\Delta v_w | b_1 = 0]\mathbb{P}[b_1 = 0] > p \cdot \kappa(1-p) - (1-p) \cdot \kappa p = 0.$$

Now, we consider the case where a^* is the winner: $\kappa b^* + (1-\kappa)c^* > \kappa p + (1-\kappa)c_1$. If a winner changed by moving an a_i^B advertiser from the set B to the set A, keeping all the drawn contextual and behavioral values fixed, then the moved advertiser must have $\kappa b_i + (1-\kappa)c_i > \kappa b^* + (1-\kappa)c^*$. Therefore, the winner's valuation can only increase in this case.

It follows that the conversion rate increases or remains the same for every advertiser we move from set B to set A. We conclude that $V_{\text{FI}}^{\text{CV}} \geq V_{\text{IA}}^{\text{CV}} \geq V_{\text{CT}}^{\text{CV}}$. ■

Proof of Proposition 8

Let's denote by $w_{\text{FI}}^{\text{IV}}, w_{\text{IA}}^{\text{IV}}, w_{\text{CT}}^{\text{IV}}$ the revenue under each information setting for an instant of auction, so that we have $W_{\text{FI}}^{\text{IV}} := \mathbb{E}[w_{\text{FI}}^{\text{IV}}], W_{\text{IA}}^{\text{IV}} := \mathbb{E}[w_{\text{IA}}^{\text{IV}}], W_{\text{CT}}^{\text{IV}} := \mathbb{E}[w_{\text{CT}}^{\text{IV}}]$. Consider an instance of auction where the contextual value of the exchange advertiser is c_1 and the contextual value of the direct advertiser is c_2. Both c_1, c_2 are drawn independently from the distribution G. First, we consider $\mathbb{E}\left[w_{\text{FI}}^{\text{IV}} | c_1, c_2\right]$, there are two cases: the case $\max\{c_1, c_2\} > \min\{c_1, c_2\} + \frac{\kappa}{1-\kappa}$, for which we find:

$$\mathbb{E}\left[w_{\text{FI}}^{\text{IV}} | c_1, c_2\right] = (\kappa + (1-\kappa)\min\{c_1, c_2\}) \cdot p + (1-\kappa)\min\{c_1, c_2\} \cdot (1-p) = \kappa p + (1-\kappa)\min\{c_1, c_2\}$$

and the case: $\min\{c_1, c_2\} + \frac{\kappa}{1-\kappa} > \max\{c_1, c_2\}$, where we have

$$\begin{aligned}\mathbb{E}\left[w_{\text{FI}}^{\text{IV}} | c_1, c_2\right] =& (\kappa + (1-\kappa)\min\{c_1, c_2\}) \cdot p^2 + (1-\kappa)\min\{c_1, c_2\} \cdot (1-p) + (1-\kappa)\max\{c_1, c_2\} \cdot (1-p)p \\ =& \kappa p^2 + (1-\kappa)\min\{c_1, c_2\} \cdot (1-p+p^2) + (1-\kappa)\max\{c_1, c_2\} \cdot (1-p)p.\end{aligned}$$

Next, we consider $\mathbb{E}\left[w_{\text{IA}}^{\text{IV}} | c_1, c_2\right]$, there are three cases: the case $c_1 > c_2 + \frac{p\kappa}{1-\kappa}$, for which we find

$$\mathbb{E}\left[w_{\text{IA}}^{\text{IV}} | c_1, c_2\right] = \kappa p + (1-\kappa)c_2$$

the case: $c_2 + \frac{p\kappa}{1-\kappa} > c_1 > c_2 - \frac{(1-p)\kappa}{1-\kappa}$, where we have

$$\mathbb{E}\left[w_{\mathrm{IA}}^{\mathrm{IV}} | c_1, c_2\right] = (\kappa p + (1-\kappa)c_2) \cdot p + (1-\kappa)c_1 \cdot (1-p),$$

and the case: $c_2 - \frac{(1-p)\kappa}{1-\kappa} > c_1$, where we have

$$\mathbb{E}\left[w_{\mathrm{IA}}^{\mathrm{IV}} | c_1, c_2\right] = (\kappa + (1-\kappa)c_1) \cdot p + (1-\kappa)c_1 \cdot (1-p) = \kappa p + (1-\kappa)c_1.$$

Lastly, we have in all cases that

$$\mathbb{E}\left[w_{\mathrm{CT}}^{\mathrm{IV}} | c_1, c_2\right] = \kappa p + (1-\kappa)\min\{c_1, c_2\}.$$

Now we can check that for all possible pairs of c_1, c_2 we have $\mathbb{E}\left[w_{\mathrm{FI}}^{\mathrm{IV}} | c_1, c_2\right] \leq \mathbb{E}\left[w_{\mathrm{IA}}^{\mathrm{IV}} | c_1, c_2\right] \leq \mathbb{E}\left[w_{\mathrm{IA}}^{\mathrm{CT}} | c_1, c_2\right]$. Taking an expectation over all possible c_1, c_2 we have that $W_{\mathrm{FI}}^{\mathrm{IV}} \leq W_{\mathrm{IA}}^{\mathrm{IV}} \leq W_{\mathrm{CT}}^{\mathrm{IV}}$ as claimed. ■

Proof of Proposition 9

From Proposition 8 we already know that for $n_1 = n_2 = 1$ we have $W_{\mathrm{FI}}^{\mathrm{IV}} \leq W_{\mathrm{CT}}^{\mathrm{IV}}$ for all $p \in [0,1]$. Now, fix $p \in [0,1]$ and consider the $n_1, n_2 > 0$ case. Note that using integration by-parts we can rewrite $W_{\mathrm{FI}}^{\mathrm{IV}}$ as:

$$\begin{aligned}
W_{\mathrm{FI}}^{\mathrm{IV}} =& (n_1+n_2)(1-\kappa)(1-p)^{n_1+n_2-1}p - (n_1+n_2)\kappa(1-p)^{n_1+n_2-1}p \\
&+ (1-\kappa)\int_0^1 \left(c - \frac{1-(1-p)G(c)}{(1-p)G'(c)}\right) d\left((1-p)G(c)\right)^{n_1+n_2} \\
&+ \int_0^1 \left(\kappa + (1-\kappa)c - \frac{(1-\kappa)(1-G(c))}{G'(c)}\right) d\left(pG(c) + (1-p)\right)^{n_1+n_2} \\
=& -(n_1+n_2)(2\kappa-1)(1-p)^{n_1+n_2-1}p + (1-\kappa)(1-p)^{n_1+n_2}\mathbb{E}_{c\sim G(c)^{n_1+n_2}}\left[c - \frac{1-(1-p)G(c)}{(1-p)G'(c)}\right] \\
&+ \mathbb{E}_{c\sim(pG(c)+(1-p))^{n_1+n_2}}\left[\kappa + (1-\kappa)c - \frac{(1-\kappa)(1-G(c))}{G'(c)}\right].
\end{aligned}$$

Where $\mathbb{E}_{c\sim F(c)}[.]$ denotes the expected value with c distributed by $F(c)$. Similarly, we can rewrite $W_{\mathrm{CT}}^{\mathrm{IV}}$ as

$$W_{\mathrm{CT}}^{\mathrm{IV}} = \int_0^1 \left(\kappa p + (1-\kappa)c - \frac{(1-\kappa)(1-G(c))}{G'(c)}\right) dG(c)^{n_1+n_2}$$

$$= \mathbb{E}_{c \sim G(c)^{n_1+n_2}} \left[\kappa p + (1-\kappa)c - \frac{(1-\kappa)(1-G(c))}{G'(c)} \right].$$

When n_1 and n_2 are large, the densities of distributions $G(c)^{n_1+n_2}$ and $(pG(c)+(1-p))^{n_1+n_2}$ become concentrated around $c = 1$. Therefore, $W_{\text{CT}}^{\text{IV}}$ tends towards $\kappa p + (1-\kappa)$. On the other hand, the first and second terms in $W_{\text{FI}}^{\text{IV}}$ tend to zero due to $(1-p)^{n_1+n_2}$ but the last term tends to $\kappa + (1-\kappa) = 1$. Hence, we have $W_{\text{FI}}^{\text{IV}} > W_{\text{CT}}^{\text{IV}}$ for all sufficiently large n_1 and n_2. ■

Proof of Lemma 3

The exchange advertisers will always bid their true valuation as it is a weakly dominant strategy to do so. Therefore, for the remainder, we will focus on the nontrivial part, which is the direct advertisers' bidding strategy.

The expected utility for a direct advertiser with contextual value c from bidding $\tilde{\beta}$ when all other $n_2 - 1$ direct advertisers follow the strategy β is given by:

$$\begin{aligned} u(\tilde{\beta};\beta,c) :=& p\Bigg[(1-\kappa)\int_0^{\max\left\{\frac{\tilde{\beta}-\kappa}{1-\kappa},0\right\}} (c-c')G(\sup\beta^{-1}[0,\kappa+(1-\kappa)c'))^{n_2-1}\left(n_1 G(c')^{n_1-1}G'(c')\right)dc' \\ &+ \int_0^{\sup\beta^{-1}[0,\tilde{\beta})} (\kappa+(1-\kappa)c-\beta(c'))G\left(\max\left\{\frac{\beta(c')-\kappa}{1-\kappa},0\right\}\right)^{n_1}\left((n_2-1)G(c')^{n_2-2}G'(c')\right)dc'\Bigg] \\ &+ (1-p)\Bigg[(1-\kappa)\int_0^{\min\left\{\frac{\tilde{\beta}}{1-\kappa},1\right\}} (c-c')G(\sup\beta^{-1}[0,(1-\kappa)c'))^{n_2-1}\left(n_1 G(c')^{n_1-1}G'(c')\right)dc' \\ &+ \int_0^{\sup\beta^{-1}[0,\tilde{\beta})} ((1-\kappa)c-\beta(c'))G\left(\min\left\{\frac{\beta(c')}{1-\kappa},1\right\}\right)^{n_1}\left((n_2-1)G(c')^{n_2-2}G'(c')\right)dc'\Bigg]. \end{aligned} \tag{5}$$

Let us restrict our attention to the bidding functions β that belong to the following class of functions:

$$\mathcal{F} := \left\{\beta \in L^1[0,1] \mid \beta \text{ is represented by a non-decreasing function } [0,1] \to [0,1]\right\}.$$

Here, $L^1[0,1]$ denotes the usual Banach space of the equivalence classes of Lebesgue-integrable functions on $[0,1]$ equipped with the usual norm $\|f\|_{L^1} := \int_0^1 |f(x)|dx$. It is not hard to verify that $\mathcal{F}$ is a convex and compact subset of $L^1[0,1]$.

We note that the sign of each of the integrals in (5) is determined by the sign of $(c-c')$, $\kappa + (1-\kappa)c - \beta(c')$, and $(1-\kappa)c - \beta(c')$, respectively, all of which are increasing functions in c

and decreasing in c'. Essentially, given β and c, finding the maximum $\tilde{\beta} = \tilde{\beta}_0$ of $u(\tilde{\beta}; \beta, c)$ is to 'integrate until the integrands are negative'. The reality is slightly more subtle, as the upper limit of each integral are different non-linear functions of $\tilde{\beta}$.

Lemma 5. *Given $\beta \in \mathcal{F}$ and $c \in [0,1]$ then $u(\tilde{\beta}; \beta, c)$ as a function of $\tilde{\beta} \in [0,1]$ achieves its global maximum inside $\{(1-\kappa)c\} \cup [\kappa, \kappa + (1-\kappa)c]$.*

Proof. First, let us observe where a maximum of $u(\tilde{\beta}; \beta, c)$ cannot be located. If $\tilde{\beta}(c) \in (\kappa + (1-\kappa)c, 1]$ then the third term of (5) is constant in $\tilde{\beta}$. The first term is strictly decreasing for $\tilde{\beta} > \kappa + (1-\kappa)c$. The second and fourth terms are non-constant if $\beta(c) > \kappa + (1-\kappa)c$ for some c, but then these two terms decrease with $\tilde{\beta}$ because $\beta(c') > \kappa + (1-\kappa)c > (1-\kappa)c$ for $c' = \max \beta^{-1}[0, \tilde{\beta}) \geq \max \beta^{-1}[0, \kappa + (1-\kappa)c)$. Similarly, if $\tilde{\beta} \in [0, (1-\kappa)c)$ then only the third and fourth terms of (5) are non-constant in $\tilde{\beta}$. The third term is strictly increasing for $\tilde{\beta} < (1-\kappa)c$ and for any $c' = \max \beta^{-1}[0, \tilde{\beta}) \leq \max \tilde{\beta}^{-1}[0, (1-\kappa)c)$, which means $\beta(c') < (1-\kappa)c$, hence the fourth term is increasing.

If $\tilde{\beta} \in [(1-\kappa)c, \kappa]$, then every term of (5) is constant except for the fourth term which could be non-constant if $\beta(c) > (1-\kappa)c$ for some c, and in that case, the fourth term is decreasing. In other words, the maximum value of $u(\tilde{\beta}; \beta, c)$ over $[(1-\kappa)c, \kappa]$ is reached at $\tilde{\beta} = (1-\kappa)c$. Since the fourth term of (5) is necessarily strictly decreasing, it is possible that $u(\tilde{\beta}; \beta, c)$ also attains its maximum value at other points in $((1-\kappa)c, \kappa]$, this fact will serve no practical implication for us.

Next, we focus on the case where $\tilde{\beta} \in [\kappa, \kappa + (1-\kappa)c]$, and we shall show that $u(\tilde{\beta}; \beta, c)$ also reaches its maximum over this interval. We note that $u(\tilde{\beta}; \beta, c)$ is left-continuous because $\sup \beta^{-1}[0, \tilde{\beta})$ is left-continuous, and the point where it is not continuous is exactly where $\{c \mid \beta(c) = \tilde{\beta}_0\}$ has non-empty interior. In particular, let $\underline{b} := \inf\{c \mid \beta(c) = \tilde{\beta}_0\}$ and $\bar{b} := \sup\{c \mid \beta(c) = \tilde{\beta}_0\}$ then it follows that $(\underline{b}, \bar{b}) \subset S(\tilde{\beta}_0)$. In that case, we have $\sup \beta^{-1}[0, \tilde{\beta}) \leq \underline{b}$ for $\tilde{\beta} \leq \tilde{\beta}_0$ and $\sup \beta^{-1}[0, \tilde{\beta}) \geq \bar{b}$ for $\tilde{\beta} > \tilde{\beta}_0$. Given that $\tilde{\beta} \in [\kappa, \kappa + (1-\kappa)c]$, the third term of (5) is constant in a neighborhood of $\tilde{\beta}_0$. Let $\delta > 0$ be arbitrarily small, then the first term of (5) will take approximately the same value at $\tilde{\beta}_0$ and at $\tilde{\beta}_0 + \delta$. If $u(\tilde{\beta}_0; \beta, c) < u(\tilde{\beta}_0 + \delta; \beta, c)$ it must be the case that the sum of the second and fourth integrals is positive over $(\underline{b}, \bar{b})$. In particular:

$$\left(\kappa + (1-\kappa)c - \tilde{\beta}_0\right) G\left(\max\left\{\frac{\tilde{\beta}_0 - \kappa}{1-\kappa}, 0\right\}\right)^{n_1} + \left((1-\kappa)c - \tilde{\beta}_0\right) G\left(\min\left\{\frac{\tilde{\beta}_0}{1-\kappa}, 1\right\}\right)^{n_1} > 0.$$

But we also know that for all $c' \in \sup \beta^{-1}[0, \tilde{\beta}_0 + \delta)$ we have $\beta(c') < \tilde{\beta}_0 + \delta$, then from the inequality above we have that the sum of the integrands of the second and fourth integrals in (5) is positive immediately to the right of $\tilde{\beta}_0$ as $\delta > 0$ is arbitrary small. Since $\tilde{\beta}_0 \leq \kappa + (1-\kappa)c$, the integrand of the first integral in (5) is also positive. It follows that $u(\tilde{\beta}; \beta, c)$ continue to increase over some right neighbourhood of $\tilde{\beta}_0 + \delta$, hence $\sup_{\tilde{\beta}} u(\tilde{\beta}; \beta, c) > \lim_{\tilde{\beta} \to \tilde{\beta}_0^+} u(\tilde{\beta}; \beta, c)$. □

Lemma 5 allows us to define the best-response set-valued function as follows:

$$BR(\beta, c) := \arg \max_{\tilde{\beta} \in [0,1]} u(\tilde{\beta}; \beta, c).$$

Let us also restrict our attention to β such that $\beta(c) \in \{(1-\kappa)c\} \cup [\kappa, \kappa + (1-\kappa)c]$.

Lemma 6. *The best-response function is closed-valued and* non-decreasing *in the sense that if* $c_1 < c_2$ *then*

$$\max BR(\beta, c_1) \leq \min BR(\beta, c_2).$$

Proof. The fact that $BR(\beta, c)$ is closed follows since according to Lemma 5, $u(\tilde{\beta}; \beta, c)$ is left-continuous and if $u(\tilde{\beta}; \beta, c)$ is discontinuous at $\tilde{\beta}_0$ then $\limsup_{\tilde{\beta} \to \tilde{\beta}_0^+} u(\tilde{\beta}; \beta, c)$ is always less than the global maximum value of u. In other words, if $\tilde{\beta}_i \in BR(\beta, c), i = 1, 2, \cdots$ and $\tilde{\beta}_i \to \tilde{\beta}_0 \in [0,1]$ then $u(\tilde{\beta}_0; \beta, c) = u(\tilde{\beta}_i; \beta, c)$ for all i, which means $\tilde{\beta}_0 \in BR(\beta, c)$. Therefore, it makes sense to talk about the maximum and minimum of $BR(\beta, c)$.

Given any $\delta > 0$, we note that it is possible to write

$$u(\tilde{\beta}; \beta, c + \delta) = u(\tilde{\beta}; \beta, c) + \Delta(\tilde{\beta}; \beta, \delta)$$

where $\Delta(\tilde{\beta}; \beta, c)$ is exactly given by (5) but with $(c - c'), (\kappa + (1-\kappa)c - \beta(c'))$, and $((1-\kappa)c - \beta(c'))$ factors replaced by $\delta, (1-\kappa)\delta$, and $(1-\kappa)\delta$, respectively. Thus, $\Delta(\tilde{\beta}; \beta, \delta)$ is a non-decreasing function in $\tilde{\beta}$ and strictly increases over $[0, 1-\kappa] \cup [\kappa, 1]$. Then the fact that $BR(\beta, c)$ is non-decreasing follows from the following elementary argument. Let $\tilde{\beta}_0 = \max BR(\beta, c)$ then $u(\tilde{\beta}_0; \beta, c) \geq u(\tilde{\beta}; \beta, c)$ for all $\tilde{\beta} \in [0, \tilde{\beta}_0)$. Therefore, $u(\tilde{\beta}_0; \beta, c+\delta) > u(\tilde{\beta}; \beta, c+\delta)$ for all $\tilde{\beta} \in [0, \tilde{\beta}_0)$ by the strict monotonicity of $\Delta(\tilde{\beta}; \beta, \delta)$, which means any other global maxima of $u(\tilde{\beta}; \beta, c+\delta)$ must be in $[\tilde{\beta}_0, \kappa + (1-\kappa)c]$, proving the lemma. □

Using Lemma 6 it is now possible to define the best-response bidding function to the bidding β of all other $n_2 - 1$ direct advertisers:

$$BR : \mathcal{F} \to \mathcal{F}, \qquad \tilde{\beta} := BR(\beta) : c \mapsto \min BR(\beta, c).$$

Where we have slightly abused the notation, using both $\tilde{\beta}$ as a particular bidding value and the bidding function, and BR as both the best response bidding set-valued function and the best response bidding function-valued map. However, we hope that any ambiguity can be resolved by context.

Lemma 7. *The best-response function BR is continuous with respect to the L^1 norm.*

We will omit the technical proof, but the intuition is clear. Any two $\beta_1, \beta_2 \in \mathcal{F}$ non-decreasing functions which are 'close' together under L^1 norm must take similar values $\beta_1(c) \approx \beta_2(c)$ at any c they are both continuous. Moreover, the location of any discontinuous points of β_1 and β_2 must be similar. The same is true for their inverses $\sup \beta_1^{-1}[0, \tilde{\beta}) \approx \beta_2^{-1}[0, \tilde{\beta})$. Hence we can expect $u(\tilde{\beta}; \beta_1, c) \approx u(\tilde{\beta}; \beta_2, c)$ for all $\tilde{\beta}$ and c and therefore the maximum point of $u(.; \beta_1, c)$ should be close to the maximum point of $u(.; \beta_2, c)$.

From Lemma 7, the response function BR is continuous with respect to L^1 norm and maps a convex compact subset $\mathcal{F} \subset L^1[0,1]$ into itself. $L^1[0,1]$ is a normed-vector space, hence it is automatically a Hausdorff locally convex topological vector space. From the Kakutani-Fan-Glicksberg Theorem, we know that BR has a fixed point. ■

A.2 Key Formulas

Unless stated otherwise, all formulas in this section are valid for any given $\kappa \in [0,1]$, $p \in [0,1]$, $n_1, n_2 \geq 0$ and an arbitrary contextual-value distribution G on $[0,1]$.

Common-value case

To deal with any discontinuities of the bidding function β we let $\beta^{-1}[a,b)$ denote the inverse image of β i.e. a set I such that $x \in I \implies \beta(x) \in [a,b)$, and $\sup \beta^{-1}[a,b)$ denotes the supremum of this set.

Advertisers' conversion rate:

The advertisers' conversion rates under each information setting are given by:

$$
\begin{aligned}
V_{\text{FI}}^{\text{CV}} =& p(n_1+n_2)\int_0^1 (\kappa+(1-\kappa)c)G(c)^{n_1+n_2-1}G'(c)dc \\
&+(1-p)(n_1+n_2)\int_0^1 (1-\kappa)cG(c)^{n_1+n_2-1}G'(c)dc,
\end{aligned}
$$

$$
\begin{aligned}
V_{\text{IA}}^{\text{CV}} =& pn_2\int_0^1 (\kappa+(1-\kappa)c)G\left(\max\left\{\frac{\beta(c)-\kappa}{1-\kappa},0\right\}\right)^{n_1} G(c)^{n_2-1}G'(c)dc \\
&+pn_1\int_0^1 (\kappa+(1-\kappa)c)G(c)^{n_1-1}G\left(\sup\beta^{-1}[0,\kappa+(1-\kappa)c)\right)^{n_2} G'(c)dc \\
&+(1-p)n_2\int_0^1 (1-\kappa)cG\left(\min\left\{\frac{\beta(c)}{1-\kappa},1\right\}\right)^{n_1} G(c)^{n_2-1}G'(c)dc \\
&+(1-p)n_1\int_0^1 (1-\kappa)cG(c)^{n_1-1}G\left(\sup\beta^{-1}[0,(1-\kappa)c)\right)^{n_2} G'(c)dc,
\end{aligned}
$$

$$
V_{\text{CT}}^{\text{CV}} = (n_1+n_2)\int_0^1 (\kappa p+(1-\kappa)c)G(c)^{n_1+n_2-1}G'(c)dc.
$$

We note that $V_{\text{FI}}^{\text{CV}} = V_{\text{CT}}^{\text{CV}}$.

Publisher's expected revenue:

The publisher's expected revenues for each information setting are given by:

$$
\begin{aligned}
W_{\text{FI}}^{\text{CV}} =& p(n_1+n_2)(n_1+n_2-1)\int_0^1 (\kappa+(1-\kappa)c)(1-G(c))G(c)^{n_1+n_2-2}G'(c)dc \\
&+(1-p)(n_1+n_2)(n_1+n_2-1)\int_0^1 (1-\kappa)c(1-G(c))G(c)^{n_1+n_2-2}G'(c)dc,
\end{aligned}
$$

$$
\begin{aligned}
W_{\text{IA}}^{\text{CV}} =& pn_1n_2\int_0^1 (\kappa+(1-\kappa)c)\left(1-G\left(\sup\beta^{-1}[0,\kappa+(1-\kappa)c)\right)\right)G(c)^{n_1-1}G\left(\sup\beta^{-1}[0,\kappa+(1-\kappa)c)\right)^{n_2-1}G'(c)dc \\
&+pn_1n_2\int_0^1 \beta(c)\left(1-G\left(\max\left\{\frac{\beta(c)-\kappa}{1-\kappa},0\right\}\right)\right)G\left(\max\left\{\frac{\beta(c)-\kappa}{1-\kappa},0\right\}\right)^{n_1-1} G(c)^{n_2-1}G'(c)dc \\
&+pn_1(n_1-1)\int_0^1 (\kappa+(1-\kappa)c)\left(1-G(c)\right)G(c)^{n_1-2}G\left(\sup\beta^{-1}[0,\kappa+(1-\kappa)c)\right)^{n_2}G'(c)dc \\
&+pn_2(n_2-1)\int_0^1 \beta(c)\left(1-G(c)\right)G\left(\max\left\{\frac{\beta(c)-\kappa}{1-\kappa},0\right\}\right)^{n_1} G(c)^{n_2-2}G'(c)dc \\
&+(1-p)n_1n_2\int_0^1 (1-\kappa)c\left(1-G\left(\sup\beta^{-1}[0,(1-\kappa)c)\right)\right)G(c)^{n_1-1}G\left(\sup\beta^{-1}[0,(1-\kappa)c)\right)^{n_2-1}G'(c)dc \\
&+(1-p)n_1n_2\int_0^1 \beta(c)\left(1-G\left(\min\left\{\frac{\beta(c)}{1-\kappa},1\right\}\right)\right)G\left(\min\left\{\frac{\beta(c)}{1-\kappa},1\right\}\right)^{n_1-1} G(c)^{n_2-1}G'(c)dc
\end{aligned}
$$

$$+ (1-p)n_1(n_1-1)\int_0^1 (1-\kappa)c\,(1-G(c))\,G(c)^{n_1-2}G\left(\sup\beta^{-1}[0,(1-\kappa)c)\right)^{n_2}G'(c)dc$$
$$+ (1-p)n_2(n_2-1)\int_0^1 \beta(c)\,(1-G(c))\,G\left(\min\left\{\frac{\beta(c)}{1-\kappa},1\right\}\right)^{n_1}G(c)^{n_2-2}G'(c)dc, \quad (6)$$

$$W_{\mathrm{CT}}^{\mathrm{CV}} = (n_1+n_2)(n_1+n_2-1)\int_0^1(\kappa p+(1-\kappa)c)(1-G(c))G(c)^{n_1+n_2-2}G'(c)dc.$$

We note that $W_{\mathrm{FI}}^{\mathrm{CV}} = W_{\mathrm{CT}}^{\mathrm{CV}}$.

Direct advertisers' payoff:

$$D_{\mathrm{FI}}^{\mathrm{CV}} = \frac{V_{\mathrm{FI}}^{\mathrm{CV}} - W_{\mathrm{FI}}^{\mathrm{CV}}}{n_1+n_2} = \int_0^1(p\kappa+(1-\kappa)c)G(c)^{n_1+n_2-1}G'(c)dc$$
$$-(n_1+n_2-1)\int_0^1(p\kappa+(1-\kappa)c)(1-G(c))G(c)^{n_1+n_2-2}G'(c)dc, \quad (7)$$

$$D_{\mathrm{IA}}^{\mathrm{CV}} = p\int_0^1(\kappa+(1-\kappa)c)G\left(\max\left\{\frac{\beta(c)-\kappa}{1-\kappa},0\right\}\right)^{n_1}G(c)^{n_2-1}G'(c)dc$$
$$+ (1-p)\int_0^1(1-\kappa)cG\left(\min\left\{\frac{\beta(c)}{1-\kappa},1\right\}\right)^{n_1}G(c)^{n_2-1}G'(c)dc$$
$$- pn_1\int_0^1(\kappa+(1-\kappa)c)\left(1-G\left(\sup\beta^{-1}[0,\kappa+(1-\kappa)c)\right)\right)G(c)^{n_1-1}G\left(\sup\beta^{-1}[0,\kappa+(1-\kappa)c)\right)^{n_2-1}G'(c)dc$$
$$- p(n_2-1)\int_0^1\beta(c)\,(1-G(c))\,G\left(\max\left\{\frac{\beta(c)-\kappa}{1-\kappa},0\right\}\right)^{n_1}G(c)^{n_2-2}G'(c)dc$$
$$- (1-p)n_1\int_0^1(1-\kappa)c\left(1-G\left(\sup\beta^{-1}[0,(1-\kappa)c)\right)\right)G(c)^{n_1-1}G\left(\sup\beta^{-1}[0,(1-\kappa)c)\right)^{n_2-1}G'(c)dc$$
$$- (1-p)(n_2-1)\int_0^1\beta(c)\,(1-G(c))\,G\left(\min\left\{\frac{\beta(c)}{1-\kappa},1\right\}\right)^{n_1}G(c)^{n_2-2}G'(c)dc, \quad (8)$$

$$D_{\mathrm{CT}}^{\mathrm{CV}} = \frac{V_{\mathrm{CT}}^{\mathrm{CV}} - W_{\mathrm{CT}}^{\mathrm{CV}}}{n_1+n_2} = D_{\mathrm{FI}}^{\mathrm{CV}}. \quad (9)$$

Exchange advertisers' payoff:

$$E_{\mathrm{FI}}^{\mathrm{CV}} = \frac{V_{\mathrm{FI}}^{\mathrm{CV}} - W_{\mathrm{FI}}^{\mathrm{CV}}}{n_1+n_2} = D_{\mathrm{FI}}^{\mathrm{CV}} = \frac{V_{\mathrm{CT}}^{\mathrm{CV}} - W_{\mathrm{CT}}^{\mathrm{CV}}}{n_1+n_2} = E_{\mathrm{CT}}^{\mathrm{CV}}, \quad (10)$$

$$E_{\mathrm{IA}}^{\mathrm{CV}} = p\int_0^1(\kappa+(1-\kappa)c)G(c)^{n_1-1}G\left(\sup\beta^{-1}[0,\kappa+(1-\kappa)c)\right)^{n_2}G'(c)dc$$
$$+ (1-p)\int_0^1(1-\kappa)cG(c)^{n_1-1}G\left(\sup\beta^{-1}[0,(1-\kappa)c)\right)^{n_2}G'(c)dc$$

$$
\begin{aligned}
&- pn_2 \int_0^1 \beta(c)\left(1 - G\left(\max\left\{\frac{\beta(c)-\kappa}{1-\kappa}, 0\right\}\right)\right) G\left(\max\left\{\frac{\beta(c)-\kappa}{1-\kappa}, 0\right\}\right)^{n_1-1} G(c)^{n_2-1} G'(c)dc \\
&- pn(n_1-1)\int_0^1 (\kappa + (1-\kappa)c)\left(1 - G(c)\right) G(c)^{n_1-2} G\left(\sup \beta^{-1}[0, \kappa + (1-\kappa)c)\right)^{n_2} G'(c)dc \\
&- (1-p)n_2 \int_0^1 \beta(c)\left(1 - G\left(\min\left\{\frac{\beta(c)}{1-\kappa}, 1\right\}\right)\right) G\left(\min\left\{\frac{\beta(c)}{1-\kappa}, 1\right\}\right)^{n_1-1} G(c)^{n_2-1} G'(c)dc \\
&- (1-p)(n_1-1)\int_0^1 (1-\kappa)c\left(1 - G(c)\right) G(c)^{n_1-2} G\left(\sup \beta^{-1}[0, (1-\kappa)c)\right)^{n_2} G'(c)dc. \quad (11)
\end{aligned}
$$

Independent-values case

In the independent-values case, all advertisers bid truthfully. An advertiser with a contextual value c but without behavioral information would bid the expected value $\kappa p + (1-\kappa)c$. An advertiser with full information would bid $v = \kappa b + (1-\kappa)c$ where v is distributed by $\tilde{G}(v) := \mathbb{P}[v' \le v] = pG\left(\max\left\{\frac{v-\kappa}{1-\kappa}, 0\right\}\right) + (1-p)G\left(\min\left\{\frac{v}{1-\kappa}, 1\right\}\right)$.

Advertisers' conversion rate:

The advertisers' conversion rates under each information setting are given by:

$$
V_{\text{FI}}^{\text{IV}} = (n_1 + n_2)\int_0^1 v\tilde{G}(v)^{n_1+n_2-1}\tilde{G}'(v)dv,
$$

$$
\begin{aligned}
V_{\text{IA}}^{\text{IV}} = n_1 &\int_0^1 v\tilde{G}(v)^{n_1-1} G\left(\min\left\{\max\left\{\frac{v-\kappa p}{1-\kappa}, 0\right\}, 1\right\}\right)^{n_2} \tilde{G}'(v)dv \\
&+ (1-p)n_2 \int_0^1 (1-\kappa)c\tilde{G}(\kappa p + (1-\kappa)c)^{n_1} G(c)^{n_2-1} G'(c)dc \\
&+ pn_2 \int_0^1 (\kappa + (1-\kappa)c)\tilde{G}(\kappa p + (1-\kappa)c)^{n_1} G(c)^{n_2-1} G'(c)dc,
\end{aligned}
$$

$$
V_{\text{CT}}^{\text{IV}} = (n_1 + n_2)\int_0^1 (\kappa p + (1-\kappa)c) G(c)^{n_1+n_2-1} G'(c)dc.
$$

Publisher's expected revenue:

The publisher's expected revenues for each information setting are given by:

$$
W_{\text{FI}}^{\text{IV}} = (n_1 + n_2)(n_1 + n_2 - 1)\int_0^1 v\left(1 - \tilde{G}(v)\right)\tilde{G}(v)^{n_1+n_2-2}\tilde{G}'(v)dv,
$$

$$
\begin{aligned}
W_{\text{IA}}^{\text{IV}} =& n_1(n_1-1)\int_0^1 v\left(1-\tilde{G}(v)\right)\tilde{G}(v)^{n_1-2}G\left(\min\left\{\max\left\{\frac{v-\kappa p}{1-\kappa},0\right\},1\right\}\right)^{n_2}\tilde{G}'(v)dv\\
&+n_1n_2\int_0^1 v\left(1-G\left(\min\left\{\max\left\{\frac{v-\kappa p}{1-\kappa},0\right\},1\right\}\right)\right)\tilde{G}(v)^{n_1-1}G\left(\min\left\{\max\left\{\frac{v-\kappa p}{1-\kappa},0\right\},1\right\}\right)^{n_2-1}\tilde{G}'(v)dv\\
&+n_1n_2\int_0^1(\kappa p+(1-\kappa)c)\left(1-\tilde{G}(\kappa p+(1-\kappa)c)\right)\tilde{G}(\kappa p+(1-\kappa)c)^{n_1-1}G(c)^{n_2-1}G'(c)dc\\
&+n_2(n_2-1)\int_0^1(\kappa p+(1-\kappa)c)\left(1-G(c)\right)\tilde{G}\left(\kappa p+(1-\kappa)c\right)^{n_1}G(c)^{n_2-2}G'(c)dc,
\end{aligned}
$$

$$
W_{\text{CT}}^{\text{IV}} = (n_1+n_2)(n_1+n_2-1)\int_0^1(\kappa p+(1-\kappa)c)(1-G(c))G(c)^{n_1+n_2-2}G'(c)dc.
$$

Direct advertisers' payoff:

$$
\begin{aligned}
D_{\text{FI}}^{\text{IV}} = \frac{V_{\text{FI}}^{\text{IV}}-W_{\text{FI}}^{\text{IV}}}{n_1+n_2} =& \int_0^1 v\tilde{G}(v)^{n_1+n_2-1}\tilde{G}'(v)dv\\
&-(n_1+n_2-1)\int_0^1 v\left(1-\tilde{G}(v)\right)\tilde{G}(v)^{n_1+n_2-2}\tilde{G}'(v)dv,
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
D_{\text{IA}}^{\text{IV}} =& (1-p)\int_0^1(1-\kappa)c\tilde{G}(\kappa p+(1-\kappa)c)^{n_1}G(c)^{n_2-1}G'(c)dc\\
&+p\int_0^1(\kappa+(1-\kappa)c)\tilde{G}(\kappa p+(1-\kappa)c)^{n_1}G(c)^{n_2-1}G'(c)dc\\
&-n_1\int_0^1 v\left(1-G\left(\min\left\{\max\left\{\frac{v-\kappa p}{1-\kappa},0\right\},1\right\}\right)\right)\tilde{G}(v)^{n_1-1}G\left(\min\left\{\max\left\{\frac{v-\kappa p}{1-\kappa},0\right\},1\right\}\right)^{n_2-1}\tilde{G}'(v)dv\\
&-(n_2-1)\int_0^1(\kappa p+(1-\kappa)c)\left(1-G(c)\right)\tilde{G}\left(\kappa p+(1-\kappa)c\right)^{n_1}G(c)^{n_2-2}G'(c)dc,
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
D_{\text{CT}}^{\text{IV}} = \frac{V_{\text{CT}}^{\text{CV}}-W_{\text{CT}}^{\text{CV}}}{n_1+n_2} =& \int_0^1(\kappa p+(1-\kappa)c)G(c)^{n_1+n_2-1}G'(c)dc\\
&-(n_1+n_2-1)\int_0^1(\kappa p+(1-\kappa)c)(1-G(c))G(c)^{n_1+n_2-2}G'(c)dc.
\end{aligned}
$$

Exchange advertisers' payoff:

$$
E_{\text{FI}}^{\text{IV}} = \frac{V_{\text{FI}}^{\text{IV}}-W_{\text{FI}}^{\text{IV}}}{n_1+n_2} = D_{\text{FI}}^{\text{IV}} = \frac{V_{\text{CT}}^{\text{IV}}-W_{\text{CT}}^{\text{IV}}}{n_1+n_2} = E_{\text{CT}}^{\text{IV}},
\tag{14}
$$

$$
\begin{aligned}
E_{\mathrm{IA}}^{\mathrm{IV}} = & \int_0^1 v\tilde{G}(v)^{n_1-1} G\left(\min\left\{\max\left\{\frac{v-\kappa p}{1-\kappa}, 0\right\}, 1\right\}\right)^{n_2} \tilde{G}'(v)dv \\
& - (n_1-1)\int_0^1 v\left(1-\tilde{G}(v)\right)\tilde{G}(v)^{n_1-2} G\left(\min\left\{\max\left\{\frac{v-\kappa p}{1-\kappa}, 0\right\}, 1\right\}\right)^{n_2} \tilde{G}'(v)dv \\
& - n_2\int_0^1 (\kappa p + (1-\kappa)c)\left(1-\tilde{G}(\kappa p + (1-\kappa)c)\right)\tilde{G}(\kappa p + (1-\kappa)c)^{n_1-1} G(c)^{n_2-1} G'(c)dc. \quad (15)
\end{aligned}
$$

A.3 First-Price Auction

A commonly used auction format in advertising auctions nowadays is the first-price auction format (see e.g., Despotakis et al., 2021). In this section, we modify our model to a first-price auction instead of a second-price auction for selling the impression, to test the robustness of our main findings.

In a first-price auction, since advertisers pay their own bid if they win, direct advertisers who do not have complete information about their valuations, are even more likely to bid conservatively (underbid) compared to a second-price auction. As a result, information asymmetry can lead to both a lower conversion rate and reduced publisher revenue, compared to the symmetric information settings, for similar reasons this happens in second-price auctions. In other words, our result that disabling microtargeting can simultaneously increase both the conversion rate and publisher revenue remains valid in common-value first-price auctions. Propositions 10 and 11 below confirm this.

Proposition 10. *For any distribution G, any $n_1, n_2 \geq 1$, $\kappa \in [0,1]$, under the common-value case, and when the auction format is first-price, we have that $V_{\mathrm{IA}}^{\mathrm{CV}} \leq V_{\mathrm{FI}}^{\mathrm{CV}} = V_{\mathrm{CT}}^{\mathrm{CV}}$.*

Proof. This result follows from Lemma 4, where the mechanism M is a first-price auction. □

Proposition 11. *For any distribution G, $n_1 > 1$, $n_2 \geq 1$, $\kappa \geq 1/2$, and sufficiently low p, under the common-value case, and when the auction format is first-price, we have that $W_{\mathrm{IA}}^{\mathrm{CV}} \leq W_{\mathrm{FI}}^{\mathrm{CV}} = W_{\mathrm{CT}}^{\mathrm{CV}}$.*

Proof. Suppose that, when there is information asymmetry, at equilibrium the direct and exchange advertisers use the bidding functions $\beta_D : [0,1] \to [0,1]$ and $\beta_E : [0,1]^2 \to [0,1]$, respectively. The expected utility of a direct advertiser with contextual value c who bids $\tilde{\beta}$ is

$$
\begin{aligned}
u_D(\tilde{\beta}; \beta_D, \beta_E, c) = & p\left(\kappa + (1-\kappa)c - \tilde{\beta}\right) G\left(\sup \beta_E^{-1}([0,\tilde{\beta}), b=1)\right)^{n_1} G(\sup \beta_D^{-1}([0,\tilde{\beta})))^{n_2-1} \\
& + (1-p)\left((1-\kappa)c - \tilde{\beta}\right) G\left(\sup \beta_E^{-1}([0,\tilde{\beta}), b=0)\right)^{n_1} G(\sup \beta_D^{-1}([0,\tilde{\beta})))^{n_2-1}.
\end{aligned}
$$

The expected utility of an exchange advertiser with contextual value c and behavioral value b who bids $\tilde{\beta}$ is

$$u_E(\tilde{\beta}; \beta_D, \beta_E, c, b) = \left(\kappa b + (1-\kappa)c - \tilde{\beta}\right) G(\sup \beta_E^{-1}([0, \tilde{\beta}), b))^{n_1 - 1} G(\sup \beta_D^{-1}([0, \tilde{\beta})))^{n_2}.$$

The expected publisher revenue is

$$\begin{aligned}
W_{\text{IA}}^{\text{CV}} =& pn_2 \int_0^1 \beta_D(c) G(\sup \beta_E^{-1}([0, \beta_D(c)), 1))^{n_1} G(c)^{n_2-1} G'(c) dc \\
&+ pn_1 \int_0^1 \beta_E(c, 1) G(c)^{n_1-1} G(\sup \beta_D^{-1}([0, \beta_E(c, 1))))^{n_2} G'(c) dc \\
&+ (1-p) n_2 \int_0^1 \beta_D(c) G(\sup \beta_E^{-1}([0, \beta_D(c)), 0))^{n_1} G(c)^{n_2-1} G'(c) dc \\
&+ (1-p) n_1 \int_0^1 \beta_E(c, 0) G(c)^{n_1-1} G(\sup \beta_D^{-1}([0, \beta_E(c, 0))))^{n_2} G'(c) dc.
\end{aligned}$$

For the other two information settings, $W_{\text{FI}}^{\text{CV}}$ and $W_{\text{CT}}^{\text{CV}}$, the revenue is identical to the second-price auction case by the revenue equivalence principle.

The conversion rate is

$$\begin{aligned}
V_{\text{IA}}^{\text{CV}} =& pn_2 \int_0^1 (\kappa + (1-\kappa)c) G\left(\sup \beta_E^{-1}([0, \beta_D(c)), 1)\right)^{n_1} G(c)^{n_2-1} G'(c) dc \\
&+ pn_1 \int_0^1 (\kappa + (1-\kappa)c) G(c)^{n_1-1} G\left(\sup \beta_D^{-1}[0, \beta_E(c, 1))\right)^{n_2} G'(c) dc \\
&+ (1-p) n_2 \int_0^1 (1-\kappa) c G\left(\sup \beta_E^{-1}([0, \beta_D(c)), 0)\right)^{n_1} G(c)^{n_2-1} G'(c) dc \\
&+ (1-p) n_1 \int_0^1 (1-\kappa) c G(c)^{n_1-1} G\left(\sup \beta_D^{-1}[0, \beta_E(c, 0))\right)^{n_2} G'(c) dc.
\end{aligned}$$

If p is sufficiently low,[17] the direct advertisers will choose to not compete with the exchange advertisers if $b = 1$ and will always bid as if $b = 0$ regardless of the actual value of b (which they do not know anyway). In this case, when $b = 0$ the game reduces to a symmetric first-price auction among all $n_1 + n_2$ advertisers, and when $b = 1$ the game reduces to a symmetric first-price auction among n_1 exchange advertisers. Consequently, we have a symmetric equilibrium where the bidding

[17] E.g. if $\kappa p + (1-\kappa) < \kappa \implies p < 2 - 1/\kappa$, which a is sufficient but not necessary bound.

functions have simple analytical closed forms as follows:

$$\beta_D(c) = (1-\kappa)\left(c - \int_0^c \left(\frac{G(t)}{G(c)}\right)^{n_1+n_2-1} dt\right),$$
$$\beta_E(c,b) = \kappa b + (1-\kappa)\left(c - \int_0^c \left(\frac{G(t)}{G(c)}\right)^{n_1+(1-b)n_2-1} dt\right). \tag{16}$$

Under the perfect-information and the contextual-targeting settings, the bidders are symmetric and independent, thus it follows from the revenue equivalence principle that $W_{\text{IA}}^{\text{CV}}$ and $W_{\text{FI}}^{\text{CV}}$ are both equal to their second-price auction counterparts, hence they are equal to each other. We also note from (16) that

$$\beta_D(c) \le \beta_E(c,b) \le \kappa b + (1-\kappa)\left(c - \int_0^c \left(\frac{G(t)}{G(c)}\right)^{n_1+n_2-1} dt\right),$$

for all c, b, where notice that the RHS is the bidding function under the perfect-information setting. The first inequality holds since $\beta_E(c,1) - \beta_D(c) \ge \kappa - (1-\kappa) \ge 0$ because $\kappa \ge 1/2$, and the second inequality holds since $\left(\frac{G(t)}{G(c)}\right)^{n_1+(1-b)n_2-1} \ge \left(\frac{G(t)}{G(c)}\right)^{n_1+n_2-1}$ for all $t, c \in [0,1]$ such that $t \le c$. In other words, the bids of all advertisers are at least as high in the perfect-information setting as under information asymmetry, hence the revenue $W_{\text{FI}}^{\text{CV}}$ is at least as high as $W_{\text{IA}}^{\text{CV}}$. $\square$