Expertise in Online Markets*

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Abstract

We examine the effect of the presence of expert buyers on other buyers, the platform, and the sellers in online markets. We model buyer expertise as the ability to accurately predict the quality, or condition, of an item, modeled as its common value. We show that nonexperts may bid more aggressively, even above their expected valuation, to compensate for their lack of information. As a consequence, we obtain two interesting implications. First, auctions with a "hard close" may generate higher revenue than those with a "soft close". Second, contrary to the linkage principle, an auction platform may obtain a higher revenue by hiding the item's common-value information from the buyers. We also consider markets where both auctions and posted prices are available and show that the presence of experts allows the sellers of high quality items to signal their quality by choosing to sell via auctions.

Keywords: competitive strategy; segmentation; pricing; sniping; online auctions; signaling

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1 Introduction

The advent of online auctions such as those in eBay led to the first massive-scale deployment of simple second-price auction mechanisms for consumer products. Even though eBay started as a platform for consumer-to-consumer auctions for selling items out of one's garage, it is now a large selling platform enabling over $200 billion commerce volume and reaching over 200 million users annually.[1] The addition of posted-price sales has fueled this growth by allowing it to serve as a competitor to other online retail sites. The growth of this new segment of online markets that combine auctions with posted prices raises important new questions about the optimal strategies for buyers and sellers as well as questions about the best design of the platform.

The eBay auction format enforces a "hard close" or ending time at which the item is sold to the highest (winning) bid. In the hours leading up to closing time, the auction is open and simulates the open outcry English auction. If all bidders had only private values, traditional auction theory dictates that the dominant strategy for every bidder is to bid up to his true value. To enable this, eBay offers a proxy bidding tool that allows a bidder to specify his maximum value, and the tool automatically bids the minimum bid increment above the current highest bid (as long as it is below the bidder-specified value). Thus, it was something of a paradox when a majority of eBay auctions exhibited sniping – the phenomenon where a bidder submits his only bid in the last few seconds of the auction, thus avoiding any response from other bidders.

While several explanations for this behavior have been advanced, one of the most intuitive and accepted ones is that of experienced bidders (Wilcox, 2000) or dealers/experts (Roth and Ockenfels, 2002). For example, Roth and Ockenfels (2002, p. 1095) argue, and provide empirical evidence, that the existence of sniping in online markets is partly due to buyers' heterogeneity in their experience with online markets and their expertise in the product category: "[T]here may be bidders who are dealers/experts and who are better able to identify high-value antiques. These well-informed bidders...may wish to bid late because other bidders will recognize that their bid is a signal that the object is unusually valuable."

In this line of reasoning, the item auctioned off is assumed to have a common value which these experts have a better knowledge of, and submitting a sniping bid is a way for experts to withhold this information to reap the advantage of this information asymmetry in the resulting price. While several papers have subsequently built upon and refined this explanation of sniping (Bajari and Hortacsu, 2003; Rasmusen, 2006; Ockenfels and Roth, 2006; Hossain, 2008; Ely and Hossain, 2009), all of them have examined the phenomenon only from the bidders' perspective. More broadly, to best of our knowledge, no other paper has studied the strategic impact of buyers' heterogeneity in expertise (which causes the sniping behavior) on the platform and sellers' strategies in online markets. In this paper, we examine the effect of the existence of expert buyers on *all* of the stakeholders in online markets: the expert and nonexpert buyers, the sellers, and the platform. We discuss the following research questions:

1. How do nonexpert buyers adjust their strategies to compete with experts?

[1] http://venturebeat.com/2013/10/16/ebay-earnings-sales-up-21-revenue-up-14-and-double-digit-paypal-user-growth/ (accessed January 2016)

2. How does the presence of experts affect the platform revenue?

3. How does the presence of experts affect the sellers' strategies in online markets?

1.1 Our Contributions

First, we show that the presence of experts encourages the nonexperts to bid more aggressively. In particular, we show that because of the sniping strategy of the expert buyers in hard-close auctions, nonexpert buyers have to bid more than their expected value; otherwise they only win items of low quality against the expert buyers. Quantifying this, we show in Proposition 1 that the higher the proportion of experts among the bidders, the more aggressively the nonexperts bid above their expected value for the item.

Next, we consider the impact of the presence of experts on the platform's strategies. In particular, should the platform maintain the hard-close format for the auction, which allows the experts to snipe, rather than switch to the "soft-close" format? Also, if the platform knows the quality value of the item and can credibly reveal it to the buyers, should it commit to sharing this information with them? We find interesting answers to these questions. Regarding the first question, at the outset, it appears that the hard-close format may hurt platform revenue since without the sniping behavior of experts, nonexpert buyers could respond to bids of experts, and the item would sell at a higher price. Since the platform's fee is usually a fixed fraction of the selling price, the platform would then have an incentive to favor the soft-close format.[2] Contrary to this expectation, we show that the aggressive bidding behavior of the nonexperts that we describe above implies that the platform's overall revenue *increases* in the hard-close format for a wide range of parameter values (Proposition 2). This is a potential new explanation as to why online auction companies such as eBay[3] retain the hard-close auction format from a revenue perspective. We note, however, that the strategic choice of soft- versus hard-close format is a complex decision affected by competition among auction platforms as well as a variety of other bidder considerations such as the avoidance of potentially costly bidding wars in hard-close auctions. Our observation above exposes a new facet in a variety of such potential explanations for the popularity of this format.

This result has another important and interesting implication regarding the second question: the platform can benefit from committing to withholding the quality information (Corollary 1). This is in contrast to the celebrated *linkage principle*[4] (Milgrom and Weber, 1982), and is driven by buyers' heterogeneity in their level of expertise. Proposition 1 can also be interpreted as a reverse *winner's curse.* In auctions with common values, bidders bid lower than their valuation to avoid the winner's curse. However, our result shows that when bidders are heterogeneous in their level of information, non-informed bidders bid more than their valuation to make up for their lack of information.

Finally, we consider the impact of the presence of expert buyers on the sellers' strategies. In particular, we investigate the choice of selling mechanisms between the auction and a

[2] In fact, some auction platforms such as the now defunct Amazon Auctions and Trademe, removed sniping by implementing a soft close that automatically extended the auction time whenever a bid is submitted.

[3] EZsniper.com provides an extensive list of auction sites with a hard close.

[4] The linkage principle argues that the auction house always benefits from committing to revealing all available information.

posted price sale when they are both available (as is common in most online auction-houses). In the presence of expert buyers, under certain conditions, we show that by selling in an auction, a seller can credibly signal[5] the quality value of his item (Proposition 3). By selling in an auction, the seller shows that he can rely on the *market* (specifically, on the expert buyers) to decide the value of the item. This is a risk that a seller with a low quality-value item cannot take. Furthermore, this signaling is possible only if there are enough experts, who know the value of the item, in the market. Otherwise, the seller of a high quality-value item will not be able to separate himself from the seller of a low-value item. In other words, the existence of experts in the market allows the sellers of high-quality products to separate themselves by selling in auctions. This finding is in line with auction houses' claim that auctions increase buyers' confidence. For example, Fraise Auction[6] argues that one of the benefits of selling in auction is that the "competitive bidding format creates confidence among the buyers when they see other people willing to pay a similar amount for the property." To best of our knowledge, this result is a new explanation for the popularity of auctions in certain product categories. We reiterate that the strategic choice of auction versus posted-price is a complex decision affected by several factors. Our observation above proposes a new explanation for why some sellers may choose to use auctions.

Taken together, we initiate the first comprehensive study of the effect of the presence of expert buyers in online markets featuring auctions with a hard close and posted prices, and establish the following results.

1. Nonexpert buyers must adjust their strategies in response to experts' sniping, and, under certain conditions, have to bid more than their expected value in hard-close auctions in equilibrium.

2. As a consequence, the platform revenue is higher in the hard-close auction than in the soft-close format for a wide range of parameter values.

3. Finally, the presence of experts in markets with hard-close auctions and posted prices allows the seller of high-quality items to credibly signal the quality of the item by selling in the auction and separating himself from sellers of low-quality items who sell using posted prices, under certain conditions.

Note that despite the explosive growth of auctions particularly in the consumer-to-consumer arena, our findings are relevant mainly to items with a significant common value component (such as collectibles, antiques, art, and used items of uncertain quality).

In what follows, we review related literature. Section 2 introduces the main model, Section 3 solves the equilibria of the model with a hard close, and Section 4 compares them with the corresponding equilibria of the auction with a soft close, which does not allow for sniping. In Section 5, we analyze the sellers' game of choosing among selling formats. We conclude the paper in Section 6. All proofs and further details are relegated to the Appendices.

[5]Note that the signal that we discuss here is the seller's choice of the selling mechanism. This is different from bids by other bidders, which can also be signals of the quality of the product.

[6]http://fraiseauction.com/why-auction/ (accessed January 2016)

1.2 Related Literature

Our work relates to the literature on online auctions with common values and a hard close, intermediaries' incentives to reveal product quality information, sellers' strategies to signal product quality, and the advantages and disadvantages of auctions versus posted prices. In the following, we review the related literature on each topic.

Bajari and Hortacsu (2003) argue that last-minute bidding is an equilibrium in a stylized model of eBay auctions with common values. They develop and estimate a structural econometric model of bidding in eBay auctions with common value and endogenous entry. Wilcox (2000) and Rasmusen (2006) use common values to model sniping and bidders' behavior on Ebay auctions. Wilcox (2000) shows that sniping increases as buyers' experience increases. Furthermore, the increase in the sniping behavior of the more experienced bidders is more pronounced for the type of items that are more likely to have a common value component. Similarly, a model with no common value as in Yoganarasimhan (2013) demonstrates no sniping behavior. Rasmusen (2006) considers a model where bidders incur a cost for learning the common value of the item. As a result, those who acquire the information snipe to hide their information from other bidders. Similar to the previous literature,[7] sniping emerges as an equilibrium strategy in our model as well. However, our focus is the effect of the presence of experts on nonexperts', sellers', and the platform's strategies and revenues, which is crucially missing in the earlier literature. Glover and Raviv (2012) show that when sellers can choose between hard-close and soft-close formats, soft close leads to a higher revenue, and experienced sellers are more likely to choose soft close. We discuss their result in Section 5, and show that soft close emerges as the unique pooling equilibrium if sellers can choose the closing format. Our result provides a new theoretical explanation for their empirical findings. In contrast to earlier work by Ockenfels and Roth (2006), who show an example in which seller revenue is lower at the equilibrium for hard-close than in the soft-close case, in our model, we show that the hard-close format increases revenue compared to the soft-close format. More specifically, we provide an explanation as to why online auction companies such as eBay retain the auction format that allows for sniping from a revenue perspective that takes into account the aggressive bidding behavior of the nonexperts.

In this paper, we show that an intermediary could benefit from withholding information about the quality of the items in an auction. This is in contrast with the well-known linkage principle by Milgrom and Weber (1982). The linkage principle argues that the auction house always benefits from committing to reveal all available information. The intuition behind the principle is that revealing the information can mitigate the winner's curse and motivates the buyers to bid more aggressively. We arrive at the contrast due to buyers' *heterogeneity* in terms of their information about the quality value of the item, as modeled by their expert status. More specifically, the result of Milgrom and Weber (1982) is established when valuation of bidders depend *symmetrically* on the unobserved signals of the other bidders, a condition that is not satisfied in our setup.[8] Withholding information, under

[7]The literature on trying to explain sniping in online auctions is vast. Other than previously mentioned papers, see also Ockenfels and Roth (2006), Hossain (2008), Wintr (2008), and Ely and Hossain (2009).

[8]Failure of the linkage principle has also been argued in a few other papers in the auction theory literature. For example, Perry and Reny (1999), Krishna (2009, Chapter 8.1), and Fang and Parreiras (2003) show the failure in setups with multiple items, ex-ante asymmetries, and budget constraints, respectively.

certain circumstances, has also been shown to increase social welfare, by Zhang (2013), in the context of product labeling. Gal-Or et al. (2007) show that, under certain conditions, a buyer benefits from withholding information in procurement schemes.

Many researchers in marketing have studied signaling unobserved quality under information asymmetry. Moorthy and Srinivasan (1995) and Soberman (2003) show that sellers can use warranties such as money-back guarantees to signal the quality of their items. Bhardwaj and Balasubramanian (2005) show that by letting the customers request information about an item, rather than revealing it without solicitation, a seller can signal the quality of his item. Mayzlin and Shin (2011) show that uninformative advertising, as an invitation for search, can be used to signal product quality. Li et al. (2009) investigate auction features such as pictures and reserve price that enable sellers to reveal more information about their credibility and product quality, and empirically examine how different types of indicators help alleviate uncertainty. Finally, Subramanian and Rao (2016) show that, by displaying daily deal sales, a platform can leverage its sales to experienced customers to signal its type and attract new customers. This is relevant to our result as in both Subramanian and Rao's paper and our paper, the existence of experts (or experienced customers) can help the sellers to extract more revenue from the nonexpert customers. However, the higher revenue is achieved using very different tools, displaying daily deal sales versus selling in auctions, in the two papers. Compared to the previous literature, we introduce a new dimension for sellers to signal the quality of their items. In particular, for product categories with a common value component where assessing the common value needs expertise (e.g., in the antiques category), we show that selling via auction can signal that the item has a high common value.

Finally, we review the related literature that compares auctions to posted price selling mechanisms. Einav et al. (2013) propose a model to explain the shift from Internet auctions to posted prices and consider two hypotheses: a shift in buyer demand away from auctions, and general narrowing of seller margins that favors posted prices. By using eBay data, they find that the former is more important. There is a significant economics literature that compares auctions to posted price mechanisms. Notably, Wang (1993) compares auctions with posted prices and shows that auctions become preferable when buyers' valuations are more dispersed. In another important paper, Bulow and Klemperer (1996) have shown that the additional revenue one can obtain by attracting one more bidder in an auction without reserve price is greater than the additional revenue by setting the optimal reserve price, hence in a sense establishing that "value of negotiating skills is small relative to value of additional competition" (p. 180). In an empirical work, Bajari et al. (2008) conclude that the choice of sales mechanism may be influenced by the characteristics of the product being sold. To the best of our knowledge, our paper is the first work that considers the signaling effects of the choice of the mechanism on buyers' beliefs. Specifically, we show that the choice of selling mechanism can be used by sellers of high-quality items as a signal of their item's quality.

2 Model

We consider a model with two buyers and one item. We assume that there are two types of buyers, *experts* and *nonexperts*, and each buyer is an expert with probability p. Given anonymity of online marketplaces, we assume that each buyer does not know whether his opponent is an expert or not.[9]

In our model, the items sold in online auctions have differing levels of "quality value," which may reflect the condition of a used good or the relative efficacy of a product among its competitors. Note that this value is similar to a common value in that its benefit accrues equally to both expert bidders (who can accurately predict quality value) and nonexpert bidders (who do not know the quality value). We assume that the quality value, denoted by a binary random variable C with realizations 0 and $c > 0$, is known only by experts and is the same for both experts and nonexperts (therefore it can be described as a common value). Moreover, the items sold in online auctions also have differing levels of "private value," which may reflect bidders' private tastes for the items, or whether they have immediate needs for the items. Each bidder may have a different private value. We assume that the private value, denoted by a binary random variable V with realizations 0 and $v > 0$, is learned privately by both experts and nonexperts.

The total value of the item for a bidder is the sum of the quality value and an additional private value component. More specifically, we assume that C has a binary distribution: $\Pr[C = c] = q$ (high common value) and $\Pr[C = 0] = 1 - q$ (low common value), also V (for each bidder) has a binary distribution: $\Pr[V = v] = r$ (high private value) and $\Pr[V = 0] = 1 - r$ (low private value). We assume that c, v, p, q and r are common knowledge. Moreover, buyers' private value types are privately known by all buyers, and the realization of C is privately known only by experts (nonexperts know only the prior probability distribution). The total value of the item for each bidder is simply $C + V$, where C is the quality value of the item and V is the buyer's specific private value.

We model the online auction with a hard close as a two-stage bidding game where the second stage represents the very last opportunity to submit a bid (the sniping window), while the first stage represents the whole window of time preceding the close. Even though in practice the period before the sniping window is a dynamic game, we model it (Stage 1) by allowing each bidder to submit a single bid: to reconcile this with reality, we can think of the highest bid that a bidder submitted before the sniping window as the first-stage bid. Bidders can observe competitors' bids of Stage 1 and respond to them in Stage 2; however, they do not have enough time to respond to competitors' bids of Stage 2. It is worth mentioning that we can derive all of our results with a more realistic dynamic game model of the first stage.[10] However, though it is a bit more involved, it does not add any further insight to our analysis, so we use the simpler two-stage formulation here.

Motivated by the fact that bidding in the sniping window has the risk of losing the bid due to erratic internet traffic, we assume that a bid in stage 2 goes through only with probability $1 - \delta$ for sufficiently small $\delta \geq 0$. Throughout the paper, we assume that $0 \leq \delta \leq \bar{\delta}$ where $\bar{\delta}$ is defined in Section A.3. This assumption implies that the risk of the bid not going through, due to δ, is not large enough to outweigh the benefit of sniping for experts. We provide an example of equilibrium structure when $\delta > \bar{\delta}$ in Section B.2. The assumption of small δ is also consistent with industry numbers that show that the rate of failure of sniping bids is

[9] On eBay and most other auction platforms, identities of bidders are revealed only after an auction ends. Furthermore, bidders can easily hide their type by creating and using a new account online.

[10] We can consider a dynamic auction in the time interval $[0, 1)$ and sniping at time 1.

less than 1%.[11]

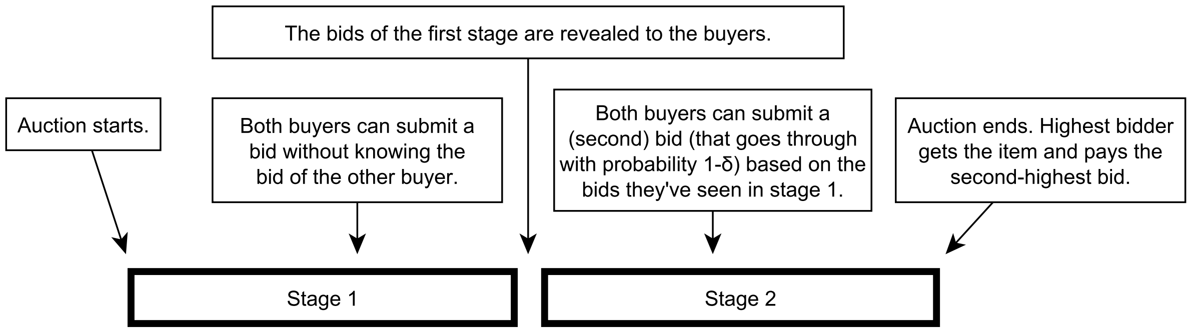

Figure 1: Timeline of the game

The timing of the model is as follows (see also Figure 1). Before Stage 1, each buyer knows his own type (expert or nonexpert), but not the type of the other buyer. If a buyer is an expert, he also knows the common value (whether $C = 0$ or $C = c$). All buyers also know their buyer-specific private values (whether $V = 0$ or $V = v$). In Stage 1, both buyers simultaneously submit their bids. After Stage 1 and before Stage 2, both buyers observe the other buyer's bid, and may be able to infer their opponent's type (and values). In Stage 2, both buyers simultaneously decide if they want to increase their bid from Stage 1, and if so by how much. In other words, bids of Stage 2 have to be greater than or equal to bids of Stage 1. Stage 2 bids are received by the auctioneer with probability $1 - \delta$. If the bid of Stage 2 is lost for a bidder (with probability δ), the auctioneer continues to use the bid of Stage 1 for that bidder. After Stage 2, the item is given to the buyer with the highest bid at the price of the second-highest bid. If there is a tie between two bidders of different values, then the item goes to the one of higher value; if both have the same value but are of different types, the tie is broken in favor of the nonexpert; if both bidders have the same value and type, the tie is broken randomly.[12]

In auctions with a soft close, there are possibly an infinite number of stages. If a bid is submitted at any stage, bidders can submit another bid in the next stage. The game ends when no bid is submitted in some stage. We also consider posted prices in Section 4. In this game, the seller posts a price z and the bidders then decide whether to buy at this price. The trade takes place at the posted price z if and only if at least one bidder is interested in the item. If both bidders want the item, each of them gets the item with probability $\frac{1}{2}$. Finally, in both types of auctions, soft and hard close, and in posted price, we assume that the platform fee is a constant fraction ξ of the selling price and is paid by the seller.

[11] For example, see https://www.quicksnipe.com/faq.php (accessed January 2016).

[12] For a full description and motivation of the tie-breaking rule, please see Section B.3. We demonstrate that our results continue to hold if we change the rule to break the tie in favor of experts rather than nonexperts.

3 Effect of Experts on Buyer Strategies

In this section, we describe the equilibria of the auction game (a formal complete treatment is in Section A.1). We derive conditions under which experts use sniping, in equilibrium, to protect their information about the common value of the item. Furthermore, we show that, under certain conditions, nonexperts with high private value bid aggressively—even above their expected valuation—to compete with experts.

We call an expert/nonexpert with high/low private value a *high/low expert/nonexpert*. Our main lemma characterizing the equilibrium (Lemma 2 in Appendix A) splits the values of v into nine ranges depending on the relative values of c, v, p, r, and q. Our characterization labels the strategies for each of the four types of players as one of five different behaviors: (i) a *sniping* strategy is adopted only by experts and involves mimicking the nonexperts in the first stage and bidding their true value only in the second stage; (ii) a *truthful* strategy involves bidding the truthful (expected) value and revising it in the second stage under any additional relevant information; (iii) an *aggressive* strategy is adopted only by high nonexperts and involves bidding over the expected value to have a chance of winning against the experts – we discuss this strategy in detail in subsection 3.2; (iv) a *mixed* strategy is a mixed version of the truthful and aggressive strategies; (v) an *underbidding* strategy is used only by low nonexperts, where they bid lower than their expected value for the item.

3.1 Experts Induce Sniping

Lemma 2 presents necessary and sufficient conditions for each of the above strategies to emerge in equilibrium for each type of bidder. In particular, we show that low experts use the sniping strategy if and only if $v \leq c \cdot \frac{(1-p)(1-q)r}{2pq(1-r)+(1-p)r}$, while high experts always use a sniping strategy.

Note that the expression $c \cdot \frac{(1-p)(1-q)r}{2pq(1-r)+(1-p)r}$ is decreasing in q and p, and increasing in r and c. In other words, a low expert's incentive to snipe increases as p or q decrease, and as r or c increase.

To see why, first note that a low expert snipes only if the common value is high. A low value of p (i.e., there are few experts in the market), a low value of q (i.e., there are few high quality items in the market), or a high value of c (i.e., quality difference between low-quality and high-quality items is large), all indicate that the low expert's information, that the common value is high, is *valuable*. This motivates the low expert to snipe and hide this information. Therefore, as p decreases, q decreases, or c increases, the threshold on v for the low expert to snipe increases. Moreover, a high value of r indicates that the opponent is likely to have a high private value. Therefore, as r increases, the probability that the low expert would win the item *without* sniping decreases, which increases his motivation to snipe. As a result, as r increases, the threshold on v for the low expert to snipe increases.

3.2 Impact of Experts on Nonexperts' Strategy

A high nonexpert's optimal strategy depends on the value of v. If v is sufficiently high ($cq + v \geq c$), a high nonexpert's expected value for the item is higher than c. In this case, high nonexperts always win the competition against low experts. For smaller values of v, the situation is more interesting. By bidding their expected value against experts, high nonexperts win only when the common value is low. Therefore, high nonexperts have to bid higher than their expected value (aggressive strategy and mixed strategy) to win a high-common-value item against low experts. Note that bidding above the expected value does not necessarily mean that they have to pay more than their expected value, because the auction format is second price. The only risk is that if two high nonexperts compete with each other, they may both bid above their expected value and end up paying more than their expected value. In this case, a nonexpert's payoff could be negative. Our first proposition discusses the conditions under which nonexperts bid more than their expected value.

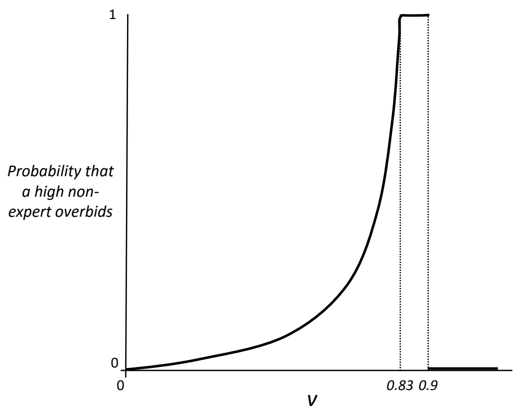

Figure 2: Probability that a high nonexpert overbids as v increases for $p = 0.3$, $r = 0.5$, $q = 0.1$, and $c = 1$.

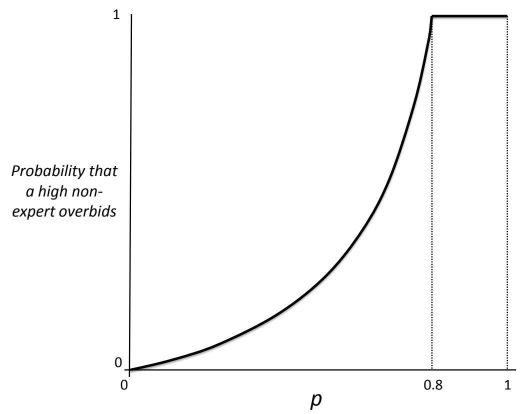

Figure 3: Probability that a high nonexpert overbids as p increases for $v = 0.5$, $r = 0.5$, $q = 0.1$, and $c = 1$.

Proposition 1. *If the expected value of a high nonexpert for the item is less than the common value of the item (i.e., $cq + v < c$), the high nonexpert may bid more than his valuation for the item in equilibrium. Moreover, the probability of overbidding increases as the fraction of experts in the market (i.e., p) increases.*

Proposition 1 shows that if the value of v is high enough, nonexperts always take the risk of over paying, and bid above their expected value in order to win against experts. However, if v is not sufficiently large, a nonexpert over bids only with some probability (depicted in Figure 2). This mixed strategy allows the nonexperts to mitigate the risk of over paying due to competition with another nonexpert. Furthermore, Proposition 1 shows that as the probability p that the opponent is an expert increases, a nonexpert's willingness to take the risk and bid above his expected value increases (depicted in Figure 3).

4 Effect of Experts on Platform Strategies

An important assumption in Proposition 1 is that experts can hide their information by sniping. The platform can eliminate sniping by extending the duration of the auction whenever a bid is submitted (this is the soft-close auction format). In this case, nonexperts always have enough time to respond to experts' bids and, therefore, do not have to bid above their expected valuation.

We show that, under certain conditions, nonexperts' aggressive behavior leads to higher revenue for the platform to the extent that the platform benefits from allowing sniping (by enforcing a hard close). In other words, experts' ability to hide their information forces the nonexperts to bid more aggressively, and ultimately leads to higher revenue for sellers and for the platform. This result also relates to platform strategies regarding the revelation of information. In Section 4.3, we show the breakdown of the *linkage principle* by showing that the platform may benefit from withholding quality information from the buyers when the buyers are heterogeneous in their level of expertise.

4.1 An Auction with a Soft Close

We now consider a model in which sniping is not possible. One way to prevent sniping is by extending the duration of the auction by a few minutes every time there is a bid near the current end time of the auction. This auction is called an auction with a soft close and was used by the now defunct Amazon Auctions. A way to model this is by starting with a game that has only one stage and every time there is a bid during the current stage, the auction extends for one more stage. In other words, every time someone makes a bid, the other buyers can see it and respond to it. In Section 4.2, we first characterize the equilibrium for a model of soft-close auctions—the details are in Lemma 3 in Section A.2. Then we compare seller's revenue and the platform's revenue across the two models. The goal is to see which ending rule results in better revenues for the sellers (and therefore for the platform).

4.2 Effect of Experts on Platform Revenue

Here we summarize the key implications of Lemma 3 that appears in Appendix A: when the soft-close format is used, high nonexperts bid their expected value. If they see a bid of c, they infer that the opponent is a low expert and the common value is high. In that case, they increase their bid to c to win the item at price c. On the other hand, with soft close, experts always reveal the value of a high-common-value item to nonexperts. This increases the nonexperts willingness to pay and in some cases leads to higher revenue for the seller. However, when there is a soft close, nonexperts do not have to bid above their valuation. This reduces the competition and can hurt sellers' revenue as well as the platform's revenue. In Lemma 1 we see that sellers can benefit from a hard close under certain conditions. We use this lemma to analyze the platform's incentive in having a hard close.

Lemma 1. *When $cq + v < c$, the seller of an item with low common value always has higher expected revenue in a hard close than in a soft close, whereas the seller of an item with high common value has higher revenue in hard than soft close if and only if p is sufficiently large.*

Lemma 1 shows that the seller of an item with low common value always benefits from a hard close. This is intuitive because a hard close causes sniping, which prevents the flow of information from experts to nonexperts. Therefore, when there is a hard close, nonexperts are more likely to overpay for an item with low common value. The interesting part is that even the seller of an item with high common value benefits from a hard close if p is high enough. This is because when there is a hard close, nonexperts know that they will not be able to infer the common value, and therefore, have to bid more aggressively to win the item. As we observe in Proposition 1, this aggressive bidding behavior increases as p increases. If p is sufficiently large, the positive effect of this aggressive bidding behavior on seller's revenue can dominate the negative effect of the lack of information flow, and result in higher revenues for the seller of a high-quality item with a hard close than with a soft close. Using the same argument, we can see that the platform can also benefit from a hard close when p is sufficiently large. This result is formalized in Proposition 2.

Proposition 2. *If the expected value of the high nonexperts for the item is less than the common value of the item (i.e., $cq + v < c$), and the fraction of experts in the market (i.e., p) is sufficiently large, the platform's revenue from a hard close is higher than that from a soft close.*

A graphical illustration of Proposition 2 is depicted in Figure 4. When $cq + v < c$ ($v/c < 0.9$ in the figure), the region where a hard close provides higher revenue appears when v is sufficiently larger than c, and p is sufficiently large. This is because higher v and higher p both lead to nonexperts' aggressive bidding, as we saw in Figures 2 and 3 and Proposition 1.

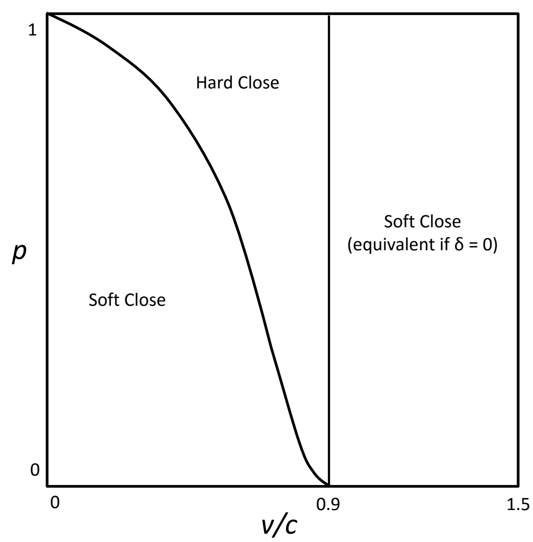

Figure 4: The regions are labeled with the format that provides higher revenue for the platform (for $r = 0.5$ and $q = 0.1$). Note that $0.9 = 1 - q$.

Proposition 2 shows that for some items the platform's revenue is higher in a hard close, while for other items the revenue is higher in a soft close. Ideally, the optimal strategy for a platform would be to use different policies for different items. However, in practice, platforms may have to use the same policy for all items for other reasons (e.g. consistent user experience). Therefore, the optimal policy will depend on the distribution of the items and the volume of the transactions across the parameter space.

4.3 Experts and the Breakdown of the Linkage Principle

Finally, we discuss the connection between the hard-close format and revelation of information in the marketplace. Note that a hard close allows the experts to protect their information about the value of the item. We know that the platform sometimes benefits from a hard close. This could suggest that the platform may also benefit from *withholding information* about the value of the item. This is an important implication because it is in contrast with the well-known "linkage principle" in auction theory (Milgrom and Weber, 1982).

The linkage principle states that auction platforms (e.g., auction houses) benefit from committing to reveal all available information about an item, positive or negative. The platform revealing the information reduces the downside risk of winning the item, also known as the winner's curse. But we show that there is also a downside in revealing the information in the presence of heterogeneous bidders, and the platform may sometimes benefit from committing to not revealing the information.

Our result shows that when bidders are *asymmetric* in terms of their information about the value of the item, bidders with less information have to bid more aggressively, otherwise, they only win the item when bidders with more information do not want the item (i.e., the common value is low). This aggressive behavior incentivizes the platform to withhold any information about the quality value of an item. This result is formalized in the following corollary.

Corollary 1. *In auctions with hard close, for medium values of p and $\frac{v}{c}$, committing to reveal the common value to the buyers decreases platform's revenue.*

We should note that the region in Figure 4 where the hard-close format provides higher revenue is the same as the region in Corollary 1 in which the platform prefers to withhold the common value information.

Our model is different from the model in Milgrom and Weber (1982) in several aspects. However, the breakdown of the linkage principle is due to only two differences in modeling assumptions. First, we allow the bidders to be heterogenous in terms of their information about the value of the item. Second, bidders do not know how much information other bidders have in this regard. We can show that even in a sealed bid second price auction, a special case of the model in Milgrom and Weber (1982), introducing these two aspects can lead to the breakdown of the linkage principle. Furthermore, both of these aspects are required for the linkage principle to break down. In particular, if bidders are asymmetric in terms of how much information they have about the value of the item, but they *know* how much information other bidders have (e.g., whether the opponent is an expert or not), Campbell and Levin (2000) establish that the linkage principle still holds.

Finally, note that Corollary 1 applies only to settings in which the platform has access to some valuable information about the item that is not easily available to all the bidders. For example, using historical market data, eBay provides a quality score for used items in certain categories. Another example is the free vehicle history reports that eBay provided for some time but later discontinued.[13]

[13] http://announcements.ebay.com/2009/11/free-vehicle-history-reports-on-ebay-motors/ (accessed January 2016)

So far we have discussed the effect of the existence of experts on nonexperts' and the platform's decisions. In Section 5, we analyze the effect of experts on sellers' choice of selling mechanism. In particular, we show that the existence of experts can help the sellers of items with a high common value to signal the value of their items to nonexperts.

5 Effect of Experts on Seller Strategies

In this section, we show that the existence of experts in the market could help the sellers to signal the quality/common value of their item to nonexperts. We look at sellers' choice of selling mechanism between an auction and a posted price sale.[14] We call the seller of an item with high common value a high-type seller, and the seller of an item with low common value a low-type seller. A seller is high-type with probability q where q is common knowledge. A seller naturally knows his own type; experts also know the seller's type (since they know the common value of items being offered). But nonexperts do not know the seller's type. We investigate whether a seller can signal his type using the selling mechanism (auction versus posted price). In particular, we derive conditions for the existence of a separating equilibrium. We show that existence of enough experts in the market is a necessary condition for a separating equilibrium to exist; furthermore, when the fraction of experts in the market, p, is sufficiently large, a separating equilibrium exists only for moderate values of $\frac{v}{c}$.

A seller sets his selling mechanism M (posted price or auction). In case of posted price, M also includes the price. For a mechanism M, we assume that all nonexperts have the same belief about a seller who uses M. In general, nonexperts' belief about a mechanism is the probability that they think a seller using that mechanism is a high type. However, since we consider only pure strategy Nash equilibria of the game, the nonexperts' belief about a mechanism is limited to three possibilities: Low (L), High (H), and Unknown (X). In belief L, nonexperts believe that a seller using mechanism M is always a low-type seller. In belief H, nonexperts believe that a seller using mechanism M is always a high-type seller. Finally, in belief X, nonexperts cannot infer anything about the seller's type and believe that the seller is high-type with probability q.

Nonexperts have beliefs about each mechanism M. In equilibrium, the beliefs must be consistent with the sellers' strategies. In particular, if both types of sellers use the same mechanism in (a pooling) equilibrium, the nonexperts' belief for that mechanism must be X. If the two types of sellers use different mechanisms in (a separating) equilibrium, the nonexperts' belief for the mechanism used by the low-type seller must be L and for the mechanism used by the high-type seller must be H. Furthermore, in an equilibrium, given the nonexperts' beliefs, sellers should not be able to benefit from changing their strategies.

Note that sniping is relevant only when the buyers' belief about some mechanism M is X. Therefore, in a separating equilibrium, the platform's decision on whether to use a soft or hard close does not affect buyers' equilibrium behavior or sellers' strategies. In other words, the following analysis applies to both soft- and hard-close cases.

In general, signaling games can have infinitely many equilibria, supported by different out-of-equilibrium beliefs in the game. Therefore, proving just the existence of an equilibrium

[14] In Section B.5, we further consider the seller's choice of closing format (hard versus soft) as a signaling mechanism.

with certain characteristics may not be a strong result. To further strengthen the support for our result that selling in auction can be used by high-type sellers as a signal of quality, we show that, under certain conditions, such an equilibrium is the only separating equilibrium that survives the "Intuitive Criterion" refinement. The Intuitive Criterion, introduced by Cho and Kreps (1987), is an equilibrium refinement that requires out-of-equilibrium beliefs to place zero weight on types that can never gain from deviating from a fixed equilibrium outcome. The Intuitive Criterion has been used in various signaling papers in the marketing literature including, but not limited to, Simester (1995), Desai and Srinivasan (1995) and Jiang et al. (2011).

Proposition 3 below shows that when the fraction of experts in the market is sufficiently large and the value of $\frac{v}{c}$ is moderate, there exists a unique separating equilibrium in which a high-type seller chooses an auction and a low-type seller chooses posted price as their respective selling mechanisms. A proof and related analysis are provided in Section A.4. Figure 5 shows the regions in which this separating equilibrium exists and is unique as a function of p and v/c.

Let us define

$$\nu_1 = \min\left(\frac{(1-p)(1-p(1-2r(1-r)))}{2r(1-r)}, \frac{(1-p)^2}{2(1-p(1-p))(1-r)r}\right),$$

$$\nu_2 = \min\left(\frac{(1-pr)^2}{r(p(2-pr)-r)}, \frac{1}{2r(1-r)}\right),$$

$$\nu_3 = \min\left(\frac{1-r}{2r}, \frac{(1-p)(2-r(1-p))}{r\left(4+(2-p(2-p))r^2-2r(3-p)\right)}\right).$$

Proposition 3. *If $\frac{v}{c} \in [\nu_1, \nu_2]$, there exists a separating equilibrium in which a high-type seller uses an auction and a low-type seller uses a posted price v. Furthermore, if $\frac{v}{c} \in (\nu_1, \nu_3)$, this is the only separating equilibrium that survives the Intuitive Criterion refinement. Finally, there exists no separating equilibrium in which a low-type seller uses an auction.*

The proof and a more elaborate discussion of Proposition 3 are relegated to Appendix A. The intuition behind the proof of Proposition 3 is as follows. First, note that in general, an auction is more favorable to a high-type than a low-type seller. This is because, in auctions, the price is determined by bidders, and expert bidders do not bid high when the seller is low-type. This allows the high-type seller to separate himself from the low-type seller by selling in an auction. But for this separating equilibrium to exist, the low-type seller's incentive to mimic has to be sufficiently low and the high-type seller's incentive to separate has to be sufficiently high. These two forces give us the thresholds ν_1 and ν_2 for existence (and ν_3 for uniqueness under IC refinement) of this equilibrium.

In a separating equilibrium, even nonexperts know that the low-type seller is a low type. Hence, nonexperts are willing to pay at most v for the item sold by the low-type seller. Therefore, the low-type seller's incentive to mimic increases as v or p decreases. If p and v are sufficiently small, since the low-type seller's incentive to mimic is sufficiently large, a separating equilibrium does not exist. This is captured by condition $\frac{v}{c} \geq \nu_1$ in Proposition 3, and is represented by the left contour in Figure 5.

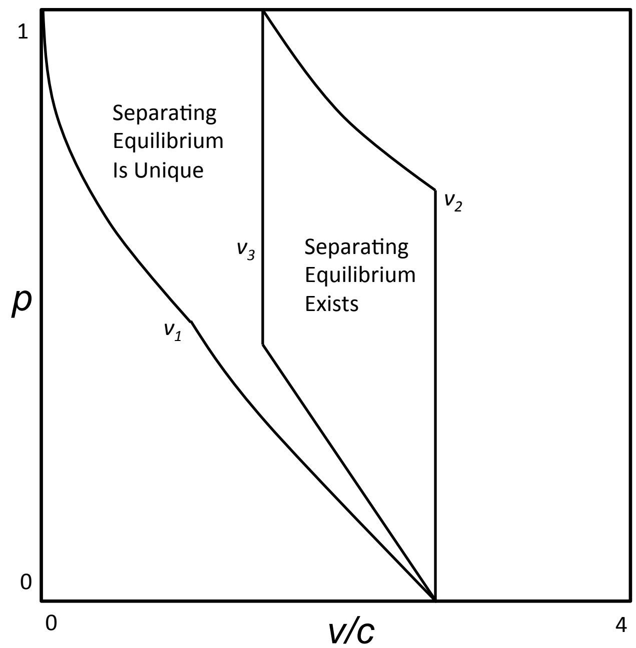

Figure 5: The graph shows the existence and uniqueness of a separating equilibrium in which the high-type seller uses auction and the low-type seller uses posted price, assuming $r = \frac{1}{4}$.

On the other hand, as $\frac{v}{c}$ increases, the common value matters less, and the high-type seller's incentive to signal his type (and to separate himself) decreases. When $\frac{v}{c}$ is large enough, we show that the high-type seller chooses to sell via an auction only if p is sufficiently small. This gives us the second condition for existence of this separating equilibrium, namely, $\frac{v}{c} \leq \nu_2$. The condition for uniqueness of the equilibrium, $\frac{v}{c} \leq \nu_3$, follows a similar intuition.

It is interesting to note that the seller's strategy in a separating equilibrium, and the conditions for existence of this equilibrium, do not depend on q. Intuitively, this is because buyers can always infer the seller's type in a separating equilibrium; therefore, when considering the seller's strategy and possible out-of-equilibrium deviations, the ex-ante probability that the seller is high type does not matter.

A Note on Hard- vs. Soft-close Formats. In this section, motivated by eBay's platform, we studied sellers' choice of auction versus posted price. It is theoretically interesting to know what happens, when limited to using auctions, if sellers can choose between hard-close and soft-close formats.[15] This is the mechanism that was employed by the now defunct Yahoo Auctions. In Section B.5, we show that if sellers can choose between soft-close and hard-close formats, the only equilibrium that survives D1 criterion refinement[16] is the one in which both types of sellers use the soft-close format (as a pure strategy pooling equilibrium). Furthermore, nonexperts' belief in the hard-close format will be low. This implies that sellers

[15]We are grateful to an anonymous referee for suggesting this question.

[16]Intuitively, D1 equilibrium refinement requires out-of-equilibrium beliefs to be supported on types that have the most to gain from deviating from a fixed equilibrium. For an extended discussion, see Fudenberg and Tirole (1991, Section 11.2).

who choose the hard-close format (out of equilibrium) will earn less revenue in expectation. Our results are consistent with the empirical findings of Glover and Raviv (2012) that show that the soft-close format leads to higher revenue than the hard-close format, and that sellers with less experience are more likely to use the hard-close format. Our explanation, however, is different from theirs, as we attribute the revenue difference to buyers' beliefs and the underlying signaling mechanism as opposed to sniping.

6 Conclusion

In this paper, we examined important questions for the buyers, sellers, and the platform of an online market supporting auctions and posted prices. We answered questions about optimal behavior for each of them using the well-documented presence of expertise among the bidders as the key underlying assumption. In particular, we studied the impact of the presence of expert bidders in online markets using a simple model of auctions with a hard close and posted prices. Motivated by large number of *used* items sold in online markets such as eBay.com, we supposed that items have differing levels of "quality" (which we model as common values), and different bidders have different capacities (which we model as expertise) to predict the quality. Bidders with low expertise may be affected by bids earlier in the auction, as these can be interpreted as signals for the quality of the item. In our model, sniping emerges as an equilibrium strategy for experts to hide their information about the quality of the item in hard-close auctions.

Our results provide several important managerial implications.

- We show that, as a consequence of sniping behavior in equilibrium by the experts in hard-close auctions, nonexpert buyers with less information have to bid aggressively, i.e., more than their expected value. This result highlights the compensatory behavior adopted by the large majority of bidders (nonexperts) that arises endogenously in these common marketplaces.

- Surprisingly, given the aggressive behavior of nonexperts, the platform's revenue can be *higher* in hard-close auctions (where sniping is prevalent) than in soft-close auctions (where sniping cannot happen). This is a new, as-yet unexplored addition to the variety of explanations of why many online auction sites use the hard-close rather than the soft-close format.

- Another interesting implication of nonexperts' aggressive behavior is that the platform can benefit in its revenue from committing to hide the information. This result has important managerial implications, as it suggests that when buyers are heterogeneous in terms of their information about the value of the item, the linkage principle does not always hold.

- When sellers can choose between auction and posted-price formats, a seller may be able to signal the high quality (or authenticity) of his item to the buyers by selling in an auction and thus separate himself from low-quality-item sellers as long as there are enough experts in the market. This provides useful guidance to vendors in such markets, where the magnitude and extent of these decisions can be moderated based on the degree and extent of the presence of expert buyers in the mix. This result also provides a new explanation for the success of auctions in categories such as antiques, art, and collectibles, where common value and therefore expertise are important.

Collectively, our work sheds light on the important differences that arise when knowledgeable or expert buyers are introduced to online marketplaces, and leads to useful guidelines for all participants in such markets.

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A Appendix

In this appendix, we present detailed explanations of the results. First, we discuss the analyses and proofs of Sections 3 and 4, in Sections A.1 and A.2, respectively. Then, we provide details of the role of the parameter δ in our model in Section A.3. Finally, in Section A.4, we detail the results of Section 5. Some of the proofs and longer discussions are relegated to Appendix B.

A.1 Analyses and Proofs of Section 3

In this section, we formally characterize the equilibria of the auction game.

Based on the relation of the parameters c, v, p, r, and q, we split the set of possible parameter values into nine mutually exclusive and collectively exhaustive ranges. In the first four ranges, we have that $cq + v < c$ and $v < cq$; in the next two, we have $cq + v < c$ and $v \geq cq$, in the next two, we have $cq + v \geq c$ and $v < cq$, and in the last range, we have $cq + v \geq c$ and $v \geq cq$.

Consider the function

$$f(c, p, r, q) = c \cdot \frac{(1-p)(1-q)r}{2pq(1-r) + (1-p)r}.$$

Let $m_1 = f(c, p, r, q), m_2 = f(c, p, 1-r, 1-q), M_1 = f(c, p, 1, q) = c \cdot (1-q)$, and $M_2 = f(c, p, 1, 1-q) = c \cdot q$. It is easy to verify that $m_1 \leq M_1$ and $m_2 \leq M_2$. We consider nine different cases as follows: $v \in [0, \min\{m_1, m_2\})$, $v \in [m_1, \min\{m_2, M_1\})$, $v \in [m_2, \min\{m_1, M_2\})$, $v \in [\max\{m_1, m_2\}, \min\{M_1, M_2\})$, $v \in [M_2, m_1)$, $v \in [\max\{m_1, M_2\}, M_1)$, $v \in [M_1, m_2)$, $v \in [\max\{m_2, M_1\}, M_2)$, and $v \in [\max\{M_1, M_2\}, +\infty)$.

To describe an equilibrium, we use the notation (s_1, s_2, s_3, s_4), which means that a high expert follows the strategy s_1, a low expert follows the strategy s_2, a high nonexpert the strategy s_3, and a low nonexpert the strategy s_4. For the bidding strategies of each type we use the following notation:

- For a high expert, consider the following strategies:

- s_1^{HE}: If $C = 0$, he bids v in the first stage and does nothing in the second stage. If $C = c$, he bids $cq + v$ in the first stage and bids $c + v$ in the second stage (sniping strategy).
- s_2^{HE}: If $C = 0$, he bids v in the first stage and does nothing in the second stage. If $C = c$, he bids c in the first stage and bids $c + v$ in the second stage (sniping strategy).

- For a low expert, consider the following strategies:
  - s^{LE}: If $C = 0$, he does nothing. If $C = c$, he bids $cq + v$ in the first stage and c in the second stage (sniping strategy).
  - t^{LE}: If $C = 0$, he does nothing. If $C = c$, he bids c in the first stage and nothing in the second stage (truthful strategy).

- For a high nonexpert, consider the following strategies:
  - x^{HNE}: He bids $cq + v$ in the first stage. If he sees a bid other than $0, v, cq$, or $cq + v$ in the first stage, he bids $c + v$ in the second stage. Otherwise, he bids c in the second stage with probability $1 - a$, where $a = 1 - \frac{2p(1-r)qv}{(1-p)r(c-(cq+v))}$ (mixed strategy).
  - o^{HNE}: He bids c in the first stage. If he sees a bid other than $0, v, cq$, or c in the first stage, he bids $c + v$ in the second stage. Otherwise, he does nothing in the second stage (aggressive strategy).
  - t^{HNE}: He bids $cq + v$ in the first stage. If he sees a bid other than $0, v, cq, c$, or $cq + v$ in the first stage, he bids $c + v$ in the second stage (truthful strategy).

- For a low nonexpert, consider the following strategies:
  - x^{LNE}: He bids v in the first stage. He bids cq in the second stage with probability $1 - g$, where $g = \frac{\frac{2pr(1-q)v}{(1-p)(1-r)(cq-v)} - \delta}{1-\delta}$ (mixed strategy).
  - u^{LNE}: He bids v in the first stage and nothing in the second stage (underbidding strategy).
  - t^{LNE}: He bids cq in the first stage and nothing in the second stage (truthful strategy).

We describe equilibrium bidding strategies for buyers in the nine cases in the following lemma.

Lemma 2. *For the auction model described in Section 2, the buyers' equilibrium bidding strategies are given below.*

1. *If $v \in [0, \min\{m_1, m_2\})$, the set of strategies $(s_1^{HE}, s^{LE}, x^{HNE}, x^{LNE})$ forms an equilibrium.*

2. *If $v \in [m_1, \min\{m_2, M_1\})$, the set of strategies $(s_2^{HE}, t^{LE}, o^{HNE}, x^{LNE})$ forms an equilibrium.*

3. *If* $v \in [m_2, \min\{m_1, M_2\})$*, the set of strategies* $(s_1^{HE}, s^{LE}, x^{HNE}, u^{LNE})$ *forms an equilibrium.*

4. *If* $v \in [\max\{m_1, m_2\}, \min\{M_1, M_2\})$*, the set of strategies* $(s_2^{HE}, t^{LE}, o^{HNE}, u^{LNE})$ *forms an equilibrium.*

5. *If* $v \in [M_2, m_1)$*, the set of strategies* $(s_1^{HE}, s^{LE}, x^{HNE}, t^{LNE})$ *forms an equilibrium.*

6. *If* $v \in [\max\{m_1, M_2\}, M_1)$*, the set of strategies* $(s_2^{HE}, t^{LE}, o^{HNE}, t^{LNE})$ *forms an equilibrium.*

7. *If* $v \in [M_1, m_2)$*, the set of strategies* $(s_1^{HE}, t^{LE}, t^{HNE}, x^{LNE})$ *forms an equilibrium.*

8. *If* $v \in [\max\{m_2, M_1\}, M_2)$*, the set of strategies* $(s_1^{HE}, t^{LE}, t^{HNE}, u^{LNE})$ *forms an equilibrium.*

9. *If* $v \in [\max\{M_1, M_2\}, +\infty)$*, the set of strategies* $(s_1^{HE}, t^{LE}, t^{HNE}, t^{LNE})$ *forms an equilibrium.*

The proof of Lemma 2 is relegated to Section B.1.

Proof of Proposition 1. This result comes directly from Lemma 2. We can see that when $m_1 \leq v < M_1$, nonexperts overbid all the time, and when $v < m_1$, they overbid with some probability. We can check in the proof of Lemma 2 that the probability of over bidding is $1 - a = \frac{2p(1-r)qv}{(1-p)r(c-(cq+v))}$. It is easy to see that this is an increasing function on p. □

A.2 Analyses and Proofs of Section 4

A.2.1 Expert strategies for soft-close auctions

As before, for the bidding strategies of each type of buyer, we use the following notation:

- For a high expert, consider the following strategy:
  - t'^{HE}: If $C = 0$, he bids v in the first stage and nothing later. If $C = c$, he bids $c + v$ in the first stage and nothing later (truthful strategy).
- For a low expert, consider the following strategy:
  - t'^{LE}: If $C = 0$, he does nothing. If $C = c$, he bids c in the first stage and nothing later (truthful strategy).
- For a high nonexpert, consider the following strategy:
  - t'^{HNE}: He bids $cq + v$ in the first stage. If he sees a bid of c or $c + v$ at some point and $cq + v < c$, he bids c in the next stage (truthful strategy).
- For a low nonexpert, consider the following strategies:
  - x'^{LNE}: He bids v in the first stage. In the second stage, he bids cq with probability $1 - w$, where $w = \frac{2pr(1-q)v}{(1-p)(1-r)(cq-v)}$, and nothing later (mixed strategy).

- u'^{LNE}: He bids v in the first stage and nothing later (underbidding strategy).
- t'^{LNE}: He bids cq in the first stage and nothing later (truthful strategy).

Lemma 3. *In a platform with soft close,*

1. *if $v \in [0, m_2)$, the set of strategies $(t'^{HE}, t'^{LE}, t'^{HNE}, x'^{LNE})$ forms an equilibrium;*
2. *if $v \in [m_2, M_2)$, the set of strategies $(t'^{HE}, t'^{LE}, t'^{HNE}, u'^{LNE})$ forms an equilibrium;*
3. *if $v \in [M_2, +\infty)$, the set of strategies $(t'^{HE}, t'^{LE}, t'^{HNE}, t'^{LNE})$ forms an equilibrium.*

Proof. With soft close, an expert is going to bid his true valuation at some point, because anything less than the true valuation will result in a lower payoff. If there is a nonexpert opponent he is going to respond to that; therefore the expert may as well bid truthfully from the first stage. More specifically, the strategies for the experts will be as follows:

- High Expert: If $C = 0$, bids v in the first stage and nothing later. If $C = c$, bids $c + v$ in the first stage and nothing later (strategy t'^{HE}).
- Low Expert: If $C = 0$, does nothing. If $C = c$, he bids c in the first stage and nothing later (strategy t'^{LE}).

For the high nonexpert, the strategy is simple as well. He will bid his expected valuation in the first stage, which is $cq + v$. If the opponent bids c or $c + v$ in the first stage (or at some later point), he will understand that he is an expert and that $C = c$, therefore if $cq + v < c$ he will bid c in the next stage (the minimum possible bid that maximizes his payoff). This is strategy t'^{HNE}.

If $cq \leq v$ (i.e. $v \geq M_2$), then a low nonexpert will bid his expected valuation in the first stage, which is cq, and then he will not do anything (strategy t'^{LNE}). Because, even if, for example, he sees a bid of c and realizes that the common value is high, by bidding c and winning the item, his payoff is still 0.

If $v < cq$ (i.e. $v < M_2$), then a low nonexpert doesn't want to bid cq from the beginning because if the opponent is a high expert and $C = 0$, he will end up with negative payoff. So, he bids v in the first stage, i.e., the maximum he can without the risk above, and waits. If he sees a bid other than v from the opponent, he will lose anyway, so it doesn't matter what strategy he will follow next, and we assume he will follow the same strategy as if he sees a bid of v. If he sees a bid of v, then he bids cq in the second stage with probability $1 - w$. No matter what happens in the second stage, he does nothing in the third stage. We need now to calculate the probability w.

First of all, if he does nothing in the second stage and he sees a bid of cq, he realizes that the opponent is another low nonexpert, but there is no reason to bid something higher because his expected payoff will be 0. If the opponent doesn't bid as well, then the auction ends, and there is no third stage. Therefore, his payoff if he sees a bid of v in the first stage and he does nothing in the second, is

$$\underbrace{\frac{pr(1-q)}{pr(1-q) + (1-p)(1-r)}(0)}_{\text{opponent is high expert and } C=0} + \underbrace{\frac{(1-p)(1-r)}{pr(1-q) + (1-p)(1-r)}\left(w\frac{cq - v}{2} + (1-w)0\right)}_{\text{opponent is low nonexpert}}.$$

If he bids cq in the second stage, his payoff is

$$\underbrace{\frac{pr(1-q)}{pr(1-q)+(1-p)(1-r)}}_{\text{opponent is high expert and } C=0}(-v) + \underbrace{\frac{(1-p)(1-r)}{pr(1-q)+(1-p)(1-r)}}_{\text{opponent is low nonexpert}}(w(cq-v)+(1-w)0).$$

We need these two expressions to be equal, from which we get

$$w = \frac{2pr(1-q)v}{(1-p)(1-r)(cq-v)}.$$

This is always non-negative, and it is < 1 iff

$$v < \frac{c(1-p)(1-r)q}{2pr(1-q)+(1-p)(1-r)} = m_2.$$

Therefore, if $v < m_2$, the low nonexpert follows the strategy x'^{LNE}.

If $v \geq \frac{c(1-p)(1-r)q}{2pr(1-q)+(1-p)(1-r)} = m_2$ (and $v < M_2$), then it is sub optimal to bid cq, therefore we set $w = 1$ (strategy u'^{LNE}). □

Proof of Lemma 1. For a low seller, a hard close is always better, because the bid of every bidder is greater than or equal to his bid when there is a soft close.

For a high seller, we know from Proposition 1 that as p increases, high nonexperts bid more and more aggressively. This makes the revenue higher as p increases, in the hard-close format. Therefore, to show the result, it is enough to show that for $p \approx 1$ the revenue with hard close is better than the revenue with soft close.

When $p \approx 1$, it holds that $m_1 \approx m_2 \approx 0$; therefore there are only two relevant equilibria in Lemma 2 (cases 4 and 6, since it is also $v < M_1$) and two in Lemma 3 (cases 2 and 3). Case 4 of Lemma 2 corresponds to case 2 of Lemma 3 and case 6 of Lemma 2 corresponds to case 3 of Lemma 3. We can see that all bids are the same in both models except the bids of the high nonexpert, which are higher with a hard close (the high nonexpert is overbidding in the equilibria 4 and 6 of Lemma 2). Therefore, overall the expected revenue is higher for a high seller with the hard-close format.

This is also illustrated in Figure 6, which shows which policy gives higher revenue to the high seller in different regions of the parameter space. Notice that this is slightly different from Figure 4, which refers to the platform's revenue.

□

Proof of Proposition 2. This result follows directly from Lemma 1. Since a low seller always benefits from a hard close, and a high seller benefits for large p, the expected platform's revenue is better with a hard close for sufficiently large p. □

The analogue of Figure 4 where the format that provides the higher revenue is labeled as a function of other parameters in the model is presented in Figure 7. In particular, in Figure 7a we can see that as r (the probability that a bidder has high private value) increases, the region where a hard close provides higher revenue becomes smaller. This is because from the perspective of a high nonexpert, high r means higher probability that the other bidder is a high nonexpert too, which in turn means lower willingness to bid aggressively in the hard-close format. This results in lower revenue for a hard close when r is large.

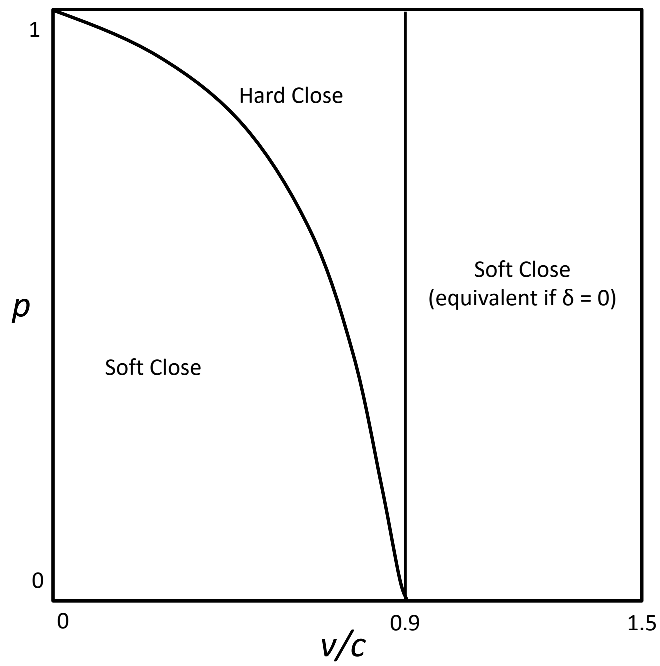


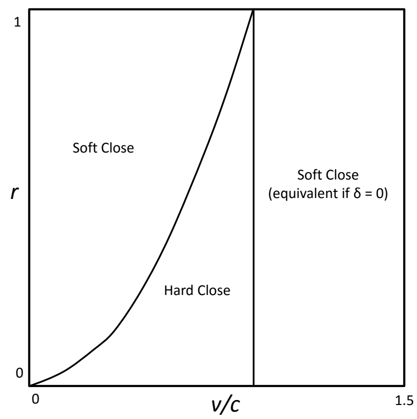
Figure 6: The regions show whether a hard close provides higher revenue for a high seller (for $r = 0.5$ and $q = 0.1$). This figure is slightly different from Figure 4 in that this compares formats that provide higher revenue for a high seller versus the earlier figure that does the same for the overall platform revenue.

Proof of Corollary 1. When the platform reveals the common value to everyone, all bidders bid their true valuation. Therefore, in the region in which the aggressive bidding of high nonexperts makes hard close better than soft close for the platform (the middle region in Figure 4), the platform prefers to hide the common value so that the high nonexperts keep bidding higher than their true valuation. □

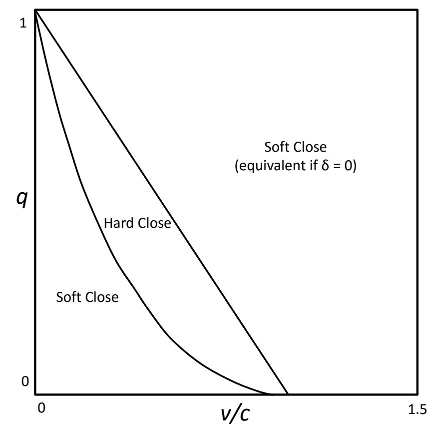
A.3 Upper-bound Condition on δ

In our model, we assume that δ is sufficiently small, i.e., $\delta \leq \bar{\delta}$. This upper-bound condition is calculated as the minimum of at most three different thresholds coming from the indifference conditions for the three of the types of players: high experts, low experts, and low nonexperts. These are the conditions that reflect the relations between the parameter values at which the current set of strategies are no longer in equilibrium. Intuitively, when $\delta > \bar{\delta}$, the cost of sniping (i.e., the risk that the bid does not go through) outweighs its benefits. Therefore, some types of bidders decide not to snipe. Since other types of bidders know this, they also have to update their strategies. As a result, we get different (and several cases of) equilibrium structures for $\delta > \bar{\delta}$. We provide an example of this in Section B.2.

A thorough discussion and calculation of the thresholds for $\bar{\delta}$ is deferred to Section B.2. The exact definition of $\bar{\delta}$ is given in Lemma 4. To provide some intuition, in Figure 8 we present plots of $\bar{\delta}$ as a function of v, of p, of q, and of r.



(a) For $p = 0.5$ and $q = 0.1$.



(b) For $r = 0.5$ and $p = 0.5$.

Figure 7: The regions are labeled with the format that provides higher revenue for the platform. This figure is an analogue of Figure 4, presenting the same result for other parameter variations.

A.4 Analyses and Proofs of Section 5

We use the following notation to explain the results of this section: Let $\pi_T^B(M)$, where $T \in \{L, H\}$ and $B \in \{L, H, X\}$ denote the expected profit of a seller who uses mechanism $M \in \{A, (B, z)\}$ (where A denotes auction, and (B, z) denotes posted price where the price is z), has type T, and nonexperts believe has type B. Let M^{pool} be the mechanism that both types of sellers use in a pooling equilibrium.

The revenue of a high- or low-type seller in an auction, where nonexperts have belief high or low, is given in the following formulas. Recall that p is the probability of being expert, and r is the probability of having high value.

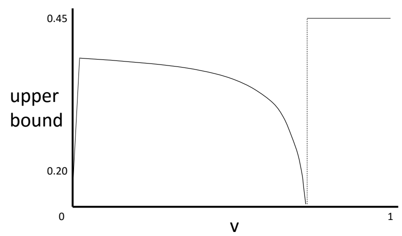
$$\pi_H^H(A) = c + r^2 v;$$

$$\pi_L^L(A) = r^2 v;$$

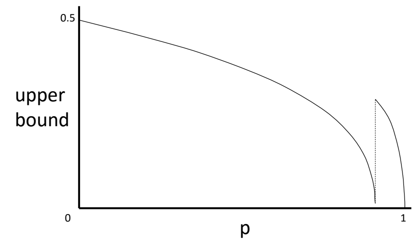
$$\pi_H^L(A) = \begin{cases} cp^2 + rv(2(1-p)p(1-r) + r) & \text{if } v \le c, \\ cp\left(2(1-p)r + p - 2(1-p)r^2\right) + r^2 v & \text{if } v > c; \end{cases}$$

$$\pi_L^H(A) = \begin{cases} c(1-p)^2 + rv(2(1-p)p(1-r) + r) & \text{if } v \le c, \\ c(1-p)(p(2(1-r)r - 1) + 1) + r^2 v & \text{if } v > c; \end{cases}$$

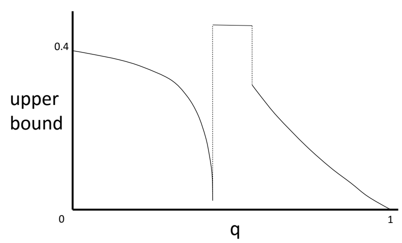
Similarly, the revenue of a high- or low-type seller using posted price with price z, in each of the four cases, is



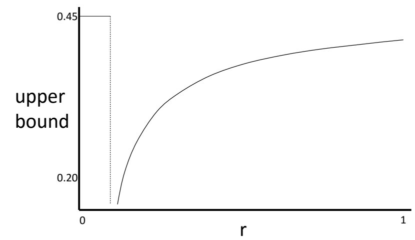
(a) As a function of v, for $c = 1$, $q = 0.1$, $r = 0.5$, $p = 0.5$.



(b) As a function of p, for $c = 1$, $q = 0.1$, $r = 0.5$, $v = 0.3$.



(c) As a function of q, for $c = 1$, $r = 0.5$, $p = 0.5$, $v = 0.3$.



(d) As a function of r, for $c = 1$, $q = 0.1$, $p = 0.5$, $v = 0.3$.

Figure 8: Plots of the upper-bound $\bar{\delta}$ as a function of v, of p, of q, and of r.

$$\pi_H^H(B, z) = \begin{cases} z & \text{if } z \leq c, \\ (2r - r^2)z & \text{if } c < z \leq c + v, \\ 0 & \text{otherwise;} \end{cases}$$

$$\pi_H^L(B, z) = \begin{cases} (1 - (1-p)^2(1-r)^2)z & \text{if } v \leq c \text{ and } z \leq v, \\ (2p - p^2)z & \text{if } v \leq c \text{ and } v < z \leq c, \\ (2pr - p^2r^2)z & \text{if } v \leq c \text{ and } c < z \leq c + v, \\ (1 - (1-p)^2(1-r)^2)z & \text{if } v > c \text{ and } z \leq c, \\ (2r - r^2)z & \text{if } v > c \text{ and } c < z \leq v, \\ (2pr - p^2r^2)z & \text{if } v > c \text{ and } v < z \leq c + v, \\ 0 & \text{otherwise;} \end{cases}$$

$$\pi_L^H(B,z) = \begin{cases} (1-p^2(1-r)^2)z & \text{if } v \le c \text{ and } z \le v, \\ (2(1-p)-(1-p)^2)z & \text{if } v \le c \text{ and } v < z \le c, \\ (2r(1-p)-r^2(1-p)^2)z & \text{if } v \le c \text{ and } c < z \le c+v, \\ (1-p^2(1-r)^2)z & \text{if } v > c \text{ and } z \le c, \\ (2r-r^2)z & \text{if } v > c \text{ and } c < z \le v, \\ (2r(1-p)-r^2(1-p)^2)z & \text{if } v > c \text{ and } v < z \le c+v, \\ 0 & \text{otherwise;} \end{cases}$$

$$\pi_L^L(B,z) = \begin{cases} (2r-r^2)z & \text{if } z \le v, \\ 0 & \text{otherwise.} \end{cases}$$

Proof of Proposition 3. We prove the proposition in three parts. In part A, we show that there is no separating equilibrium in which the high-type seller uses posted price and the low-type seller uses auction. In part B, we show that when $v \in [\nu_1, \nu_2]$, there exists a separating equilibrium in which the high-type seller uses auction and the low-type seller uses posted price v. Finally, in part C, we show that for $v \in (\nu_1, \nu_3)$, this is the only equilibrium that survives the Intuitive Criterion refinement.

Part A. Note that $\pi_L^L(A) < \pi_L^L(B,v)$, which means that conditioned on the type of sellers being revealed, the low-type seller always prefers posted price v to auction. Therefore, the low-type seller never uses an auction in a separating equilibrium.

Part B. Note that for a separating equilibrium in which the high-type seller uses auction and the low-type seller uses posted price to exist, the following two conditions are necessary and sufficient:

$$\pi_L^L(B,z) \le \pi_L^H(A),$$
$$\pi_H^H(A) \ge \pi_H^L(B,z).$$

The first condition guarantees that the low-type seller cannot benefit from deviating and the second condition guarantees that the high-type seller cannot benefit from deviating. $\pi_L^L(B,z)$ is optimized at $z=v$, and is equal to $(2r-r^2)v$. Having this less than or equal to $\pi_L^H(A)$, and using basic calculus, gives us the condition $\frac{v}{c} \ge \nu_1$. Similarly, solving the second inequality for v gives us condition $\frac{v}{c} \le \nu_2$. If nonexpert buyers' beliefs are L for posted prices and H for auction, then $\nu_1 \le \frac{v}{c} \le \nu_2$ is also sufficient for existence of this equilibrium.

Part C. Finally, we show that if $\nu_1 \le \frac{v}{c} \le \nu_3$, the separating equilibrium in which the high-type seller uses auction and the low-type seller uses posted price is the only pure strategy separating Nash equilibrium that survives the Intuitive Criterion refinement. Assume for sake of contradiction that there exists another separating equilibrium. We already know from Part A of this proof that the low-type seller cannot be using auction. Therefore, both types must be using posted price (with different prices) in this equilibrium. Using the same argument as in Part B of the proof, we know that the low-type seller must be using posted price v. Suppose that the high-type seller is using posted price ζ. For this to be a separating equilibrium, the low-type seller should not benefit from deviating and mimicking the high-type seller: $\pi_L^L(B,v) \ge \pi_L^H(B,\zeta)$. Using basic calculus, we can show that this implies the following condition on ζ. We must have $\zeta \leq \frac{(r-2)v}{(p-1)(pr-r+2)}$. Let $\pi^* = \pi_H^H(B, \zeta)$ be the profit of the high-type seller (in the hypothetical separating equilibrium) subject to this constraint.

If $\pi_H^H(A) > \pi^*$, then the high-type seller benefits from deviating to auction unless nonexperts' belief about auction is L. But note that if $\frac{v}{c} > \nu_1$, nonexperts' belief about auction cannot be L according to the Intuitive Criterion refinement. Specifically, since the high-type seller benefits from deviating to auction and the low-type seller never benefits from deviating to auction even if buyers' belief in auction is H, according to the Intuitive Criterion refinement, buyers' belief in auction should be H. Therefore, if $\pi_H^H(A) > \pi^*$ the high-type seller benefits from deviating to auction and the hypothetical equilibrium cannot exist. Using basic calculus, the condition $\pi_H^H(A) > \pi^*$ reduces to $\frac{v}{c} \leq \nu_3$. Therefore, for $\frac{v}{c} \in (\nu_1, \nu_3)$, the separating equilibrium in which the high-type seller uses auction and the low-type seller uses posted price is the only pure strategy separating Nash equilibrium that survives the Intuitive Criterion refinement. □

B Additional Appendix

In this appendix, we first give a complete proof of the equilibrium-characterization lemma 2 in Section B.1. In Section B.2, we calculate the explicit upper bound $\bar{\delta}$ on the value of δ alluded to in Section A.3. We then provide a robustness check on the choice of our tie-breaking rule in Section B.3 by showing that the analog of the main lemma 2 continues to hold even if we invert the tie-breaking rule to favor experts instead of nonexperts. In the next Section B.4, we show an extrapolation of our main result for the case when the private value distribution has a support of more than two values, thus lending support for our main observations in the limiting continuous case. Finally, in Section B.5, we consider equilibria when sellers sell in an auction but can choose between hard-close versus soft-close formats.

B.1 Proof of Lemma 2

Proof. We will group the nine equilibria into four cases. These are $v < \min\{M_1, M_2\}$ (equilibria 1, 2, 3 and 4), $M_2 \leq v < M_1$ (equilibria 5 and 6), $\max\{M_2, M_1\} \leq v$ (equilibrium 9), and $M_1 \leq v < M_2$ (equilibria 7 and 8).

Case 1. First, assume that $v < \min\{M_1, M_2\}$. This means that $cq + v < c$ and $v < cq$. Consider the following general set of strategies:

- High Expert: If $C = 0$, bids v in the first stage and does nothing in the second stage. If $C = c$, bids $cq + v$ in the first stage and bids $c + v$ in the second stage.
- Low Expert: If $C = 0$, does nothing. If $C = c$, he bids $cq + v$ in the first stage and c in the second stage.
- High nonexpert: Bids $cq + v$ in the first stage. If he sees a bid other than $0, v, cq$, or $cq + v$ in the first stage, he bids $c + v$ in the second stage. Otherwise, he bids c in the second stage with probability $1 - a$.
- Low nonexpert: Bids cq in the first stage with probability b and v with probability $1 - b$. If his bid was v, he bids cq in the second stage with probability $1 - g$.

The probabilities a, b, g are as yet undetermined . For now, we just assume that $a > 0$. We will examine if anyone has an incentive to change his strategy and at the same time try to determine the probabilities and the conditions for which the above is an equilibrium. These conditions will give us the proof that equilibria 1 and 3 are correct. Later, we will relax the assumption on a and examine what happens when $a = 0$; this will lead us to conditions that equilibria 2 and 4 are correct, and will conclude the proof of the four equilibria in the first case.

— High Expert with $C = 0$: His valuation is v and now he bids v in the first stage. If he does nothing in the first stage and he bids v in the second stage, then there is some probability that his bid will not go through with a payoff of 0, and in the case it goes through, his payoff would be the same in all cases as if he had bid v in the first stage (against a low nonexpert, his payoff is 0 in both cases). Therefore, it is optimal for him to follow this strategy.

— High Expert with $C = c$: His valuation is $c+v$ and now he bids $cq+v$ in the first stage and bids $c + v$ in the second stage. We consider three alternative strategies which dominate all the rest, and we prove that he doesn't have any incentive to deviate to any of them.

One strategy is to bid v in the first stage and $c + v$ in the second stage. This strategy has a different result for him only if his bid in the second stage doesn't go through. In that case, by having a bid of v instead of a bid of $cq + v$ can only decrease his payoff.

Another strategy is to bid 0 in the first stage and $c + v$ in the second. The behavior of the rest of the bidders will not change, but his payoff will decrease because he can lose in some cases whereas the bid of $cq + v$ would give him a positive payoff.

The last strategy is to bid $c + v$ (or c) in the first stage and nothing (or $c + v$) in the second. However, if we assume that his bid will go through in the second stage, with the alternative strategy the result would be the same in all cases except in the case he faces a high nonexpert, where his payoff strictly decreases. Therefore, since δ is sufficiently small[17], it is better to bid in the second stage.

— Low Expert with $C = 0$: His value is 0 and he does nothing, which is optimal for him.

— Low Expert with $C = c$: His value is c. The payoff if he bids c in the first stage is

$$\begin{aligned}A(\delta) = \quad & \underbrace{pr((1-\delta)0 + \delta(c - (cq+v)))}_{\text{opponent is high expert}} \\ & + \underbrace{p(1-r)((1-\delta)0 + \delta(c - (cq+v)))}_{\text{opponent is low expert}} \\ & + \underbrace{(1-p)r((1-\delta)0 + \delta(c - (cq+v)))}_{\text{opponent is high nonexpert}} \\ & + \underbrace{(1-p)(1-r)(b(c-cq) + (1-b)(g(c-v) + (1-g)(1-\delta)(c-cq) + (1-g)\delta(c-v)))}_{\text{opponent is low nonexpert}}.\end{aligned}$$

The payoff with the current strategy is

$$\begin{aligned}B(\delta) = \quad & \underbrace{(1-\delta)}_{\text{bid goes through}} \cdot \Bigg[\underbrace{pr((1-\delta)0 + \delta(c - (cq+v)))}_{\text{opponent is high expert}} \\ & + \underbrace{p(1-r)((1-\delta)0 + \delta(c - (cq+v)))}_{\text{opponent is low expert}}\end{aligned}$$

[17] Formally, this condition means $\delta < \bar{\delta}$, which we discuss in Section B.2.

$$+ \underbrace{(1-p)r(a(c-(cq+v)) + (1-a)(1-\delta)0 + (1-a)\delta(c-(cq+v)))}_{\text{opponent is high nonexpert}}$$

$$\left. + \underbrace{(1-p)(1-r)(b(c-cq) + (1-b)(g(c-v) + (1-g)(1-\delta)(c-cq) + (1-g)\delta(c-v)))}_{\text{opponent is low nonexpert}} \right]$$

$$+ \underbrace{\delta}_{\text{bid doesn't go through}} \cdot \left[\underbrace{pr((1-\delta)0 + \delta 0)}_{\text{opponent is high expert}} \right.$$

$$+ \underbrace{p(1-r)\left((1-\delta)0 + \delta\frac{c-(cq+v)}{2}\right)}_{\text{opponent is low expert}}$$

$$+ \underbrace{(1-p)r(0)}_{\text{opponent is high nonexpert}}$$

$$\left. + \underbrace{(1-p)(1-r)(b(c-cq) + (1-b)(g(c-v) + (1-g)(1-\delta)(c-cq) + (1-g)\delta(c-v)))}_{\text{opponent is low nonexpert}} \right].$$

It holds that $B(0) - A(0) = (1-p)ra(c-(cq+v)) > 0$ (for $a > 0$), which means that for sufficiently small δ, $B(\delta) > A(\delta)$, i.e. the current strategy is better.

The alternative is to bid something else in the first stage other than $cq+v$ or c, and c in the second, but this doesn't increase the payoff.

— High nonexpert: His expected valuation is $cq+v$. Bidding something else other than $cq+v$ in the first stage will not change the bidding behavior of the opponent to something better for him, therefore he prefers to bid $cq+v$ in the first stage rather than wait.

In the second stage, it doesn't matter what they do if they see a bid of 0 or v or cq, since the result cannot change. If they see a bid other than $0, v, cq$, or $cq+v$ (like c or $c+v$), something that doesn't happen in the equilibrium, they assume that the common value is high which means that their valuation is $c+v$, so they bid $c+v$. The reason is that the only one who might have incentive to deviate from the current strategies is an expert with $C = c$ who tries to bluff in some way to hide the common value.

If they see a bid of $cq+v$, then they know that their opponent is a high expert with $C = c$, or a low expert with $C = c$, or a high nonexpert. Their payoff by doing nothing in the second stage is

$$A_2 = \underbrace{\frac{prq}{prq + p(1-r)q + (1-p)r}((1-\delta)0 + \delta(c+v-(cq+v)))}_{\text{opponent is high expert and } C=c}$$

$$+ \underbrace{\frac{p(1-r)q}{prq + p(1-r)q + (1-p)r}((1-\delta)0 + \delta(c+v-(cq+v)))}_{\text{opponent is low expert and } C=c}$$

$$+ \underbrace{\frac{(1-p)r}{prq + p(1-r)q + (1-p)r}(a0 + (1-a)(1-\delta)0 + (1-a)\delta 0)}_{\text{opponent is high nonexpert}},$$

while their payoff by bidding c is

$$B_2 = \underbrace{(1-\delta)}_{\text{bid goes through}} \cdot \left[\underbrace{\frac{prq}{prq + p(1-r)q + (1-p)r}((1-\delta)0 + \delta(c+v-(cq+v)))}_{\text{opponent is high expert and } C=c} \right.$$

$$+ \underbrace{\frac{p(1-r)q}{prq+p(1-r)q+(1-p)r}((1-\delta)v+\delta(c+v-(cq+v)))}_{\text{opponent is low expert and } C=c}$$

$$\left. + \underbrace{\frac{(1-p)r}{prq+p(1-r)q+(1-p)r}(a0+(1-a)(1-\delta)\frac{cq+v-c}{2}+(1-a)\delta 0)}_{\text{opponent is high nonexpert}} \right]$$

$$+ \underbrace{\delta}_{\text{bid doesn't go through}} \cdot A_2.$$

By bidding $c+\epsilon$ for some small $\epsilon > 0$, his payoff can only decrease. By bidding $c-\epsilon$, the payoff is the same as if they stay with the bid of $cq+v$ (according to the tie-breaking rule, if two bidders are both high, the nonexpert wins). It holds that

$$B_2 - A_2 = (1-\delta)\left[\frac{p(1-r)q}{prq+p(1-r)q+(1-p)r}(1-\delta)v + \frac{(1-p)r}{prq+p(1-r)q+(1-p)r}(1-a)(1-\delta)\frac{cq+v-c}{2}\right],$$

and we want this to be equal to 0 to permit mixing these strategies, which will give us an expression for the mixing probability a. This is

$$a = 1 - \frac{2p(1-r)qv}{(1-p)r(c-(cq+v))}.$$

This is always ≤ 1. We assumed also that $a > 0$, which is equivalent to $v < \frac{c(1-p)r(1-q)}{2p(1-r)q+(1-p)r} = m_1$. Therefore, we need this condition to have an equilibrium in this case.

If $1 - \frac{2p(1-r)qv}{(1-p)r(c-(cq+v))} \leq 0$, which is equivalent to $v \geq m_1$ and corresponds to $a = 0$, we need a different set of strategies and we consider this case later.

— Low nonexpert: His expected valuation is cq. His payoff if he bids cq in the first stage is

$$\begin{aligned} A_3 = \; & \underbrace{pr(q0+(1-q)(-v))}_{\text{opponent is high expert}} \\ & + \underbrace{p(1-r)(q0+(1-q)0)}_{\text{opponent is low expert}} \\ & + \underbrace{(1-p)r(0)}_{\text{opponent is high nonexpert}} \\ & + \underbrace{(1-p)(1-r)(b0+(1-b)(g(cq-v)+(1-g)(1-\delta)0+(1-g)\delta(cq-v)))}_{\text{opponent is low nonexpert}} \end{aligned}$$

His payoff if he bids v in the first stage and follows the current strategy in the second stage is

$$\begin{aligned} B_3 = \; & \underbrace{pr(q0+(1-q)(g0+(1-g)(1-\delta)(-v)+(1-g)\delta 0))}_{\text{opponent is high expert}} \\ & + \underbrace{p(1-r)(q0+(1-q)0)}_{\text{opponent is low expert}} \\ & + \underbrace{(1-p)r(0)}_{\text{opponent is high nonexpert}} \\ & + (1-p)(1-r)\Bigg[\underbrace{b0+(1-b)(g^2(\frac{cq-v}{2})+(1-g)g((1-\delta)(cq-v)+\delta\frac{cq-v}{2})}_{\text{opponent is low nonexpert}} \\ & + g(1-g)((1-\delta)0+\delta\frac{cq-v}{2})+(1-g)^2((1-\delta)^2 0+(1-\delta)\delta(cq-v)+\delta(1-\delta)0+\delta^2\frac{cq-v}{2}))\Bigg]. \end{aligned}$$

Now, in the second stage, if a low nonexpert with a bid of v sees any bid other than v from the opponent, bidding cq or nothing in the second stage doesn't affect his payoff. If he sees a bid of v, then he knows that the opponent is either a high expert with $C = 0$ or a low nonexpert. If he does nothing in the second stage, his payoff is

$$\begin{aligned} A_4 = \quad & \underbrace{\frac{pr(1-q)}{pr(1-q)+(1-p)(1-r)(1-b)}}_{\text{opponent is high expert and } C=0}(0) \\ & + \frac{(1-p)(1-r)(1-b)}{pr(1-q)+(1-p)(1-r)(1-b)}\underbrace{\left(g\frac{cq-v}{2} + (1-g)(1-\delta)0 + (1-g)\delta\frac{cq-v}{2}\right)}_{\text{opponent is low nonexpert}}, \end{aligned}$$

while if he bids cq, the payoff is

$$\begin{aligned} B_4 = \quad & \underbrace{(1-\delta)}_{\text{bid goes through}} \Bigg[\underbrace{\frac{pr(1-q)}{pr(1-q)+(1-p)(1-r)(1-b)}(-v)}_{\text{opponent is high expert and } C=0} \\ & + \frac{(1-p)(1-r)(1-b)}{pr(1-q)+(1-p)(1-r)(1-b)}\underbrace{(g(cq-v) + (1-g)(1-\delta)0 + (1-g)\delta(cq-v))}_{\text{opponent is low nonexpert}} \Bigg] \\ & + \underbrace{\delta A_4}_{\text{bid doesn't go through}}. \end{aligned}$$

It must hold that $A_4 = B_4$ to permit mixing these strategies, from which we get an expression for the mixing probability g which is

$$g = \frac{\frac{2pr(1-q)v}{(1-p)(1-r)(1-b)(cq-v)} - \delta}{1-\delta}.$$

This expression is non-negative for sufficiently small δ and it is < 1 iff

$$v < \frac{c(1-p)(1-r)(1-b)q}{2pr(1-q)+(1-p)(1-r)(1-b)}.$$

For $b = 0$ and the corresponding g, we get $A_3 \leq B_3$ (for $g < 1$), therefore the current strategy of the low expert is optimal and we get an equilibrium. For this reason, we set $b = 0$. The above condition then becomes

$$v < \frac{c(1-p)(1-r)q}{2pr(1-q)+(1-p)(1-r)} = m_2.$$

If $v \geq m_2$, then we set $g = 1$ (which corresponds to strategy u^{LNE}).

This ends the proof for equilibria 1 and 3.

When $a = 0$, the strategy for the low expert we considered above is not always optimal. This happens when $v \geq m_1$. More specifically, since he knows that the high nonexpert will bid c in the second stage for sure, he has no reason to wait until the second stage to bid, and bids c from the first stage. With the same logic, since a high nonexpert knows for sure that he will bid c in the second stage, it is even better to bid c from the first stage. Moreover, when a high nonexpert sees a bid of c in the first stage, he doesn't know for sure what the opponent is, so he doesn't increase his bid. This will change also the strategy for the high expert with $C = c$. In the first stage, he prefers to bid c instead of $cq + v$, because a bid of $cq + v$ would reveal that he is a high expert and $C = c$. So, the equilibrium when $v \geq \frac{c(1-p)r(1-q)}{2p(1-r)q+(1-p)r} = m_1$ is as follows.

- High Expert: If $C = 0$, bids v in the first stage and does nothing in the second stage. If $C = c$, bids c in the first stage and bids $c + v$ in the second stage.

- Low Expert: If $C = 0$, does nothing. If $C = c$, he bids c in the first stage and nothing in the second stage.

- High nonexpert: Bids c in the first stage. If he sees a bid other than $0, v, cq$ or c in the first stage, he bids $c + v$ in the second stage. Otherwise, he does nothing in the second stage.

- Low nonexpert: Bids v in the first stage. He bids cq in the second stage with probability $1 - g$.

The proofs for the high expert, the low expert and the low nonexpert are the same. We need to check if the high nonexpert has any reason to change strategy. An alternative strategy for him would be the one he had before, i.e. to bid $cq + v$ in the first stage and c in the second with some probability. So, suppose that he had bidden $cq + v$ in the first stage and he sees a bid of c. His payoff by doing nothing in the second is 0, while the payoff to bid c in the second stage is

$$
\begin{aligned}
B' = \quad & \underbrace{(1-\delta)}_{\text{bid goes through}} \cdot \Bigg[\underbrace{\frac{prq}{prq + p(1-r)q + (1-p)r}((1-\delta)0 + \delta(c + v - (c)))}_{\text{opponent is high expert and } C=c} \\
& + \underbrace{\frac{p(1-r)q}{prq + p(1-r)q + (1-p)r}(c + v - c)}_{\text{opponent is low expert and } C=c} \\
& + \underbrace{\frac{(1-p)r}{prq + p(1-r)q + (1-p)r}(\frac{cq + v - c}{2})}_{\text{opponent is high nonexpert}} \Bigg] \\
& + \underbrace{\delta}_{\text{bid doesn't go through}} \cdot 0.
\end{aligned}
$$

This is ≥ 0 for $v \geq \frac{c(1-p)r(1-q)}{2prq\delta+2p(1-r)q+(1-p)r}$, which is true since

$$
v > \frac{c(1-p)r(1-q)}{2p(1-r)q + (1-p)r} \geq \frac{c(1-p)r(1-q)}{2prq\delta + 2p(1-r)q + (1-p)r}.
$$

Therefore, he is better off by bidding c rather than 0 in the second stage. This means that by bidding in the first stage he can increase his payoff. All other possible strategies are trivially dominated by those we considered above.

This ends the proof for equilibria 2 and 4.

Summarizing the first case, when $a > 0$ (i.e. $v < m_1$) and $g < 1$ (i.e. $v < m_2$), we get the equilibrium $(s_1^{HE}, s^{LE}, x^{HNE}, x^{LNE})$, when $a = 0$ (i.e. $v \geq m_1$) and $g < 1$ (i.e. $v < m_2$), we get the equilibrium $(s_2^{HE}, t^{LE}, o^{HNE}, x^{LNE})$, when $a > 0$ (i.e. $v < m_1$) and $g = 1$ (i.e. $v \geq m_2$), we get the equilibrium $(s_1^{HE}, s^{LE}, x^{HNE}, u^{LNE})$, and when $a = 0$ (i.e. $v \geq m_1$) and $g = 1$ (i.e. $v \geq m_2$), we get the equilibrium $(s_2^{HE}, t^{LE}, o^{HNE}, u^{LNE})$.

Case 2. Assume now that $M_2 \leq v < M_1$. This means that $cq \leq v$ and $cq + v < c$. Consider the following set of strategies:

- High Expert: If $C = 0$, bids v in the first stage and does nothing in the second stage. If $C = c$, bids $cq + v$ in the first stage and bids $c + v$ in the second stage.
- Low Expert: If $C = 0$, does nothing. If $C = c$, he bids $cq + v$ in the first stage and c in the second stage.
- High nonexpert: Bids $cq + v$ in the first stage. If he sees a bid other than $0, v, cq$, or $cq + v$ in the first stage, he bids $c + v$ in the second stage. Otherwise, he bids c in the second stage with probability $1 - a$.
- Low nonexpert: Bids cq in the first stage and nothing in the second.

We now investigate if anyone has incentive to change strategy. For $a > 0$, the arguments for all types of bidders are the same as in the previous case except for the low nonexpert.

The expected valuation of a low nonexpert is cq. Now he bids cq in the first stage and his expected payoff is 0. The only way to get the item is only if he faces another low nonexpert, in which case they both bid cq and there is a tie. But even in this case he has to pay cq, so his payoff is 0. He cannot achieve a better payoff, since it is never optimal to bid something above his expected valuation.

This ends the proof for equilibrium 5.

Similarly as in the previous case, the equilibrium when $v \geq \frac{c(1-p)r(1-q)}{2p(1-r)q+(1-p)r} = m_1$ (which means $a = 0$) is as follows.

- High Expert: If $C = 0$, bids v in the first stage and does nothing in the second stage. If $C = c$, bids c in the first stage and bids $c + v$ in the second stage.
- Low Expert: If $C = 0$, does nothing. If $C = c$, he bids c in the first stage and nothing in the second stage.
- High nonexpert: Bids c in the first stage. If he sees a bid other than $0, v, cq$, or c in the first stage, he bids $c + v$ in the second stage. Otherwise, he does nothing in the second stage.
- Low nonexpert: Bids cq in the first stage and nothing in the second.

This ends the proof for equilibrium 6.

Summarising the second case, when $a > 0$ (i.e. $v < m_1$), we get the equilibrium $(s_1^{HE}, s^{LE}, x^{HNE}, t^{LNE})$, and when $a = 0$ (i.e. $v \geq m_1$), we get the equilibrium $(s_2^{HE}, t^{LE}, o^{HNE}, t^{LNE})$.

Case 3. Next, suppose that $\max\{M_2, M_1\} \leq v$. This means that $\max\{cq, c(1-q)\} \leq v$. We consider the following set of strategies:

- High Expert: If $C = 0$, bids v in the first stage and does nothing in the second stage. If $C = c$, bids $cq + v$ in the first stage and bids $c + v$ in the second stage.
- Low Expert: If $C = 0$, does nothing. If $C = c$, he bids c in the first stage and nothing in the second stage.
- High nonexpert: Bids $cq + v$ in the first stage. If he sees a bid other than $0, v, cq, c,$, or $cq + v$ in the first stage, he bids $c + v$ in the second stage.
- Low nonexpert: Bids cq in the first stage and nothing in the second.

This is the simplest case. Both high and low nonexperts have nothing to lose by bidding their expected valuation, therefore they do so from the first stage. The low nonexpert has no reason to hide his identity, therefore he bids his valuation from the first stage. The same is true for a high expert with $C = 0$. Finally, the high expert with $C = c$ bids the highest possible he can in the first stage without revealing that he is a high expert, which is a bid of $cq + v$, and then he bids $c + v$ in the second stage. If he bids $c + v$ from the first stage, then his payoff strictly decreases because of the possibility that the opponent is a high nonexpert.

This ends the proof for equilibrium 9.

Summarizing the third case, we get the equilibrium $(s_1^{HE}, t^{LE}, t^{HNE}, t^{LNE})$.

Case 4. Finally, suppose that $M_1 \leq v < M_2$. This means that $c(1 - q) \leq v < cq$. We consider two cases:

- If $v < \frac{c(1-p)(1-r)q}{2pr(1-q)+(1-p)(1-r)} = m_2$, the following is an equilibrium.
  - High Expert: If $C = 0$, bids v in the first stage and does nothing in the second stage. If $C = c$, bids $cq + v$ in the first stage and bids $c + v$ in the second stage.
  - Low Expert: If $C = 0$, does nothing. If $C = c$, he bids c in the first stage and nothing in the second stage.
  - High nonexpert: Bids $cq+v$ in the first stage. If he sees a bid other than $0, v, cq, c,$, or $cq + v$ in the first stage, he bids $c + v$ in the second stage.
  - Low nonexpert: Bids v in the first stage. He bids cq in the second stage with probability $1 - g$, where $g = \frac{\frac{2pr(1-q)v}{(1-p)(1-r)(cq-v)} - \delta}{1-\delta}$.
- If $v \geq \frac{c(1-p)(1-r)q}{2pr(1-q)+(1-p)(1-r)} = m_2$, the following is an equilibrium.
  - High Expert: If $C = 0$, bids v in the first stage and does nothing in the second stage. If $C = c$, bids $cq + v$ in the first stage and bids $c + v$ in the second stage.
  - Low Expert: If $C = 0$, does nothing. If $C = c$, he bids c in the first stage and nothing in the second stage.
  - High nonexpert: Bids $cq+v$ in the first stage. If he sees a bid other than $0, v, cq, c,$, or $cq + v$ in the first stage, he bids $c + v$ in the second stage.
  - Low nonexpert: Bids v in the first stage and nothing in the second.

For the experts and the high nonexpert, the proofs are similar to the previous case. For the low nonexpert, the proof is similar to the second case.

This ends the proof for equilibria 7 and 8.

Summarizing the fourth case, when $g < 1$ (i.e. $v < m_2$), we get the equilibrium ($s_1^{HE}, t^{LE}, t^{HNE}, x^{LNE}$), and when $g = 1$ (i.e. $v \geq m_2$), we get the equilibrium ($s_1^{HE}, t^{LE}, t^{HNE}, u^{LNE}$).

□

B.2 More on δ

Recall that δ is the probability that a bid in the second stage does not go through (due to network or other technical difficulties). For the result of Lemma 2 to hold, in the model section, we assumed that $\delta \leq \bar{\delta}$. In this section, we elaborate on how to calculate the value of $\bar{\delta}$. We also briefly discuss how the equilibrium structure changes when $\delta > \bar{\delta}$.

We start with the first case of Lemma 2, i.e. when $v \leq \min\{m_1, m_2\}$. In that case, the set of strategies ($s_1^{HE}, s_1^{LE}, x^{HNE}, x^{LNE}$) is an equilibrium for sufficiently small δ.

More specifically, there are three threshold values τ_1, τ_2, τ_3, and the case 1 of Lemma 2 holds if $\delta \leq \min\{\tau_1, \tau_2, \tau_3\}$. The first threshold, τ_1, corresponds to the strategy of the high expert. When δ exceeds this threshold, a high expert with $C = c$ prefers to bid $c + v$ in the first stage instead of waiting to bid in the second stage (i.e. instead of following strategy s_1^{HE}).

The second threshold, τ_2, corresponds to the strategy of the low expert. When δ exceeds this threshold, a low expert with $C = c$ prefers to bid c in the first stage instead of following the strategy s_1^{LE}. To compute τ_2, we have to find the minimum δ for which $B(\delta) \geq A(\delta)$ in the proof of Lemma 2, or equivalently solve the equation $B(\delta) = A(\delta)$ for δ.

The third threshold, τ_3, corresponds to the strategy of the low nonexpert. When δ exceeds this threshold, a low nonexpert prefers to bid cq in the first round, i.e. prefers to follow the strategy t^{LNE} instead of the strategy x^{LNE}. To compute τ_3, we have to find the minimum δ for which the probability g in the proof of Lemma 2 is non-negative, or equivalently solve the equation $g(\delta) = 0$ for δ.

The closed-form expressions for the three thresholds are given below.

$$\tau_1 = \frac{-\sqrt{2c^2(p-2)(p-1)(q-1)^2r^2 + 4c(p-2)p(q-1)q(r-1)rv + p^2(r-1)^2v^2} + 2c(p-1)(q-1)r + p(4q-1)(r-1)v}{cp(q-1)r + 2p(2q-1)(r-1)v},$$

$$\tau_2 = \max\left\{ \frac{1}{\frac{(p(r-1)-2r)(c(q-1)+v)}{\sqrt{2}\sqrt{(p(r-1)-2r)(c(q-1)+v)(c(p-1)(q-1)r+v(p(2q(r-1)+r)-r))}} + 1}, \frac{1}{-\frac{(p(r-1)-2r)(c(q-1)+v)}{\sqrt{2}\sqrt{(p(r-1)-2r)(c(q-1)+v)(c(p-1)(q-1)r+v(p(2q(r-1)+r)-r))}} + 1} \right\} =$$

$$= \frac{1}{\frac{(p(r-1)-2r)(c(q-1)+v)}{\sqrt{2}\sqrt{(p(r-1)-2r)(c(q-1)+v)(c(p-1)(q-1)r+v(p(2q(r-1)+r)-r))}} + 1},$$

$$\tau_3 = \frac{2pr(1-q)v}{(1-p)(1-r)(cq+v)}.$$

The plot in Figure 9 shows how platform's revenue changes as δ increases in the interval $[0, \min\{\tau_1, \tau_2, \tau_3\}]$.

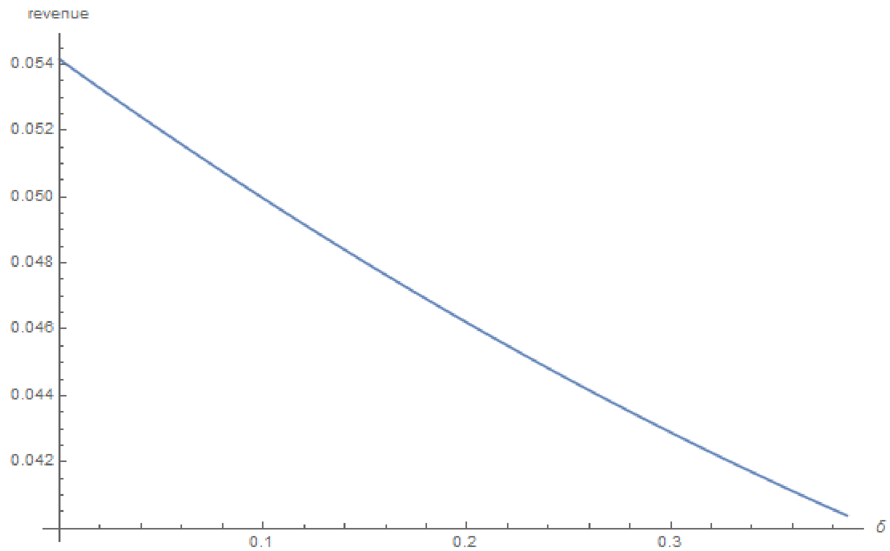


Figure 9: Platform's revenue as δ increases, for $v = 0.03$, $c = 1$, $p = 0.5$, $r = 0.5$, and $q = 0.1$. It is $\tau_1 = 0.445287$, $\tau_2 = 0.386605$, and $\tau_3 = 0.415385$.

We can see that as δ increases, platform's revenue decreases. The reason for this is that the equilibrium remains the same, i.e. the bids of the buyers are the same, but the probability that some bids don't go through increases, therefore the expected final price of the item is lower.

We continue with the second case of Lemma 2, where the equilibrium is $(s_2^{HE}, t^{LE}, o^{HNE}, x^{LNE})$. Here we need only two bounds for δ, one for the high expert and one for the low nonexpert, since the low expert bids only in the first round independently of the value of δ.

The threshold for the low nonexpert remains the same as in the previous case, i.e. it is τ_3. However, the bound for the high expert will change due to the change in the strategies of the other bidders.

Consider the following strategy for the high expert:

- t^{HE}: If $C = 0$, he bids v in the first stage and does nothing in the second stage. If $C = c$, he bids $c + v$ in the first stage and nothing in the second stage (truthful strategy).

To find the new threshold for the high expert, we need to compare his payoff when he uses the strategy s_2^{HE}, his payoff when he uses the strategy t^{HE}, and see when the first is larger than the second, which will make $(s_2^{HE}, t^{LE}, o^{HNE}, x^{LNE})$ an equilibrium. The new threshold is

$$\sigma_1 = \max\left\{\frac{2p+\sqrt{2}\sqrt{(p-2)(p-1)}-2}{p}, \frac{2p-\sqrt{2}\sqrt{(p-2)(p-1)}-2}{p}\right\} = \\ = \frac{2p+\sqrt{2}\sqrt{(p-2)(p-1)}-2}{p}.$$

Therefore, the necessary and sufficient condition for the second case of Lemma 2 is $\delta \leq \min\{\sigma_1, \tau_3\}$. Similarly for every case of the lemma, we can find the bound for δ. The following result summarizes all the cases.

Lemma 4. *The necessary and sufficient condition for δ in Lemma 2 is $\delta \leq \bar{\delta}$, where*

$$\bar{\delta} = \begin{cases} \min\{\tau_1, \tau_2, \tau_3\}, & \text{if } v \in [0, \min\{m_1, m_2\}) \text{ (case 1)}, \\ \min\{\sigma_1, \tau_3\}, & \text{if } v \in [m_1, m_2) \text{ (cases 2, 7)}, \\ \min\{\tau_1, \tau_2\}, & \text{if } v \in [m_2, m_1) \text{ (cases 3,5)}, \\ \sigma_1, & \text{if } v \in [\max\{m_1, m_2\}, +\infty) \text{ (cases 4,6,8,9)}. \end{cases}$$

B.2.1 An example of equilibrium for $\delta > \bar{\delta}$.

Since, according to the industry numbers, the probability that the sniping bid does not go through is less than 1%, in the main model we only consider the case in which δ is relatively small. However, it is theoretically interesting to know what happens for larger values of δ. Depending on the value of δ and other parameters in the model, the full analysis leads to too many cases the discussion of which is beyond the scope of this paper. However, to gain some intuition, in the following we discuss one example of the equilibrium structure for $\delta > \bar{\delta}$. Interestingly, we see that the platform's revenue could be non-monotone in δ.

Consider the following parameter values, $c = 1$, $q = 0.1$, $r = 0.5$, $p = 0.5$, $v = 0.7$, and $\delta = 0.44$. We have $v > M_2 = 0.1$ and $v < m_1 = 0.75$; therefore, we are in case 5 of Lemma 2. However, it holds that $\tau_1 = 0.348355$ and $\tau_2 = 0.257284$, i.e., δ exceeds the necessary and sufficient threshold given in Lemma 4 for $(s_1^{HE}, s^{LE}, x^{HNE}, t^{LNE})$ to be an equilibrium.

In particular, the low expert does not want to snipe since δ is larger than τ_2, so he moves his bid to the first stage. In other words, he prefers to follow the strategy t^{LE} instead of the strategy s^{LE}. This will cause a change in the strategies of the other bidders. The new equilibrium will be $(s_2^{HE}, t^{LE}, o^{HNE}, t^{LNE})$, which happens to be the same as case 6 of Lemma 2.

This is because if we cosnider the proof of case 6, the requirements for the equilibrium are satisfied, even though $v < m_1$. First, it holds that $\delta < \sigma_1 = 0.44949$, so the high expert's best response is s_2^{HE}. Moreover, it is the case that $v > \frac{c(1-p)r(1-q)}{2prq\delta+2p(1-r)q+(1-p)r} = 0.698758$, which makes o^{HNE} the best response for the high nonexpert. The value of v for the low expert and the low nonexpert doesn't matter as long as the others follow the aforementioned strategies. Therefore, $(s_2^{HE}, t^{LE}, o^{HNE}, t^{LNE})$ is an equilibrium.

This causes the following interesting phenomenon. Even though in the interval $[0, 0.257284]$, platform's revenue is a decreasing function of δ, as depicted in Figure 9; as δ increases more, outside this interval, platform's revenue increases. In this example, for $\delta = 0.257284$, platform's revenue is 0.251106, while for $\delta = 0.44$, it is 0.264497.

This is mainly because the low expert has changed his strategy by moving his bid of c from the second stage to the first stage, i.e., from the point where his bid was not going through with probability $\delta = 0.257284$ to the point where his bid always goes through (when $\delta = 0.44$); this change has a positive effect on the expected price of the item.

As δ increases even more (above σ_1), we see similar patterns: intervals in which platform's revenue is a decreasing function of δ, and some 'jumps' of the revenue in between due to the change of strategies by bidders. In the limit, when $\delta \approx 1$, no-one will bid in the second stage and the auction will be like a sealed-bid second price auction where everyone bids in the first stage.

B.3 Choice of Tie-breaking Rule

In this section, first we elaborate on the choice of our tie-breaking rule. We argue that this rule always favors the bidder who is willing to bid slightly higher than the current bid (i.e. his payoff continues to remain positive if he slightly raises his bid) which the other bidder is not able to match. Then, we show that our results are robust to the choice of the tie-breaking rule. In particular, we show that the equilibrium strategies remain almost unchanged, and our main results continue to hold, under a very different tie-breaking rule.

Recall that we use the following tie-breaking rule: If there is a tie between a low-type bidder and a high-type bidder, then the item goes to the high-type. If the two bidders are of the same type but of different expertise levels, then the item goes to the nonexpert. Finally, if the two bidders are of the same type and of the same expertise level, then the winner is determined by a fair coin toss.

There are two interesting cases for which the tie-breaking rule has an effect in the equilibria described in the main lemma. The first is when a high nonexpert faces a low expert who knows that $C = c$, and they both bid c. In this case, the high nonexpert is willing to bid above c to win the tie and take the item, because his valuation is higher than c, but the low expert cannot do the same since his valuation is c. Therefore, the tie-breaking rule favors the bidder who would be willing to pay a slightly higher price.

The second case is when a high expert who knows that $C = 0$ faces a low nonexpert, and they both bid v. In this case, the low nonexpert does not want to win the item, because his valuation is below v. Therefore, he has an incentive to bid a bit below v, whereas the high expert does not want to do the same since his valuation is v. Thus, again the tie-breaking rule favors the bidder who has higher willingness to pay.

Intuitively, if we break the tie in favor of the other bidder in any of the above cases, one of the bidders would want to increase or decrease his bid by the smallest possible amount $\epsilon > 0$. Since in our model the strategy space is continuous and not discrete, such ϵ does not exist. We use this tie-breaking rule to avoid such complications. However, to further demonstrate the robustness of our results, in the following, we show that equilibrium strategies, and therefore all of our main results, continue to hold if we change the rule in the opposite direction and favor the experts over the nonexperts.

B.3.1 Changing the tie-breaking rule.

Consider the following alternate tie-breaking rule: If there is a tie between two bidders of different expertise levels, then the item goes to the expert. Otherwise, the winner is determined by a fair coin toss. To reduce the number of cases in the analysis, we assume that $\delta = 0$, and only focus on the bids of the second stage.

As explained above, this game does not have a pure strategy Nash equilibrium unless we discretize the bidding space. We show that as the size of the discretization step converges to zero (i.e., the bidding space converges to continuous), the equilibrium outcome of the new tie-breaking rule converges to that of the old tie-breaking rule.

To discretize the strategy space, we assume that bidders can bid c or $c+\epsilon$, for some very small $\epsilon > 0$, but they cannot bid anything in between. This assumption will come into play when there is a tie between a low expert and a high nonexpert who both bid c (the first case discussed earlier in this section). Note that without this discretization, high nonexperts sometimes want to bid the smallest number strictly larger than c; this is because high nonexperts want to win against experts but lose against other high nonexperts. We assume that the rest of the strategy space remains unchanged.

We start by defining the strategies for the different types of bidders.

- For a high expert, consider the following strategy:
  - t^{HE}: If $C = 0$, he bids v. If $C = c$, he bids $c + v$.
- For a low expert, consider the following strategy:
  - t^{LE}: If $C = 0$, he does nothing. If $C = c$, he bids c.
- For a high nonexpert, consider the following strategies:
  - x^{HNE}: He bids $cq + v$ with probability a and $c + \epsilon$ with probability $1 - a$, where $a := a(\epsilon) = 1 - \frac{2p(1-r)qv}{(1-p)r(c+\epsilon-(cq+v))}$.
  - o^{HNE}: He bids $c + \epsilon$.
  - t^{HNE}: He bids $cq + v$.
- For a low nonexpert, consider the following strategies:
  - x^{LNE}: He bids v with probability g and cq with probability $1 - g$, where $g = \frac{2pr(1-q)v}{(1-p)(1-r)(cq-v)}$.
  - u^{LNE}: He bids v.
  - t^{LNE}: He bids cq.

We define also a new threshold for v, the analogous of the old m_1, that now depends also on ϵ. We have that $m_1 := m_1(\epsilon) = \frac{(c+\epsilon-cq)(1-p)r}{2p(1-r)q+(1-p)r}$.

Intuitively, high nonexperts who want to over-bid now bid $c + \epsilon$ instead of c. This allows them to win against experts, even though ties are broken in favor of experts. We now describe the equilibrium bidding strategies for buyers in nine cases in the following lemma.

Lemma 5. *For the auction model described above with the alternative tie-breaking rule, the buyers' equilibrium bidding strategies are given below.*

1. *If $v \in [0, \min\{m_1(\epsilon), m_2\})$, the set of strategies $(t^{HE}, t^{LE}, x^{HNE}, x^{LNE})$ forms an equilibrium.*

2. *If $v \in [m_1, \min\{m_2, M_1\})$, the set of strategies $(t^{HE}, t^{LE}, o^{HNE}, x^{LNE})$ forms an equilibrium.*

3. *If $v \in [m_2, \min\{m_1(\epsilon), M_2\})$, the set of strategies $(t^{HE}, t^{LE}, x^{HNE}, u^{LNE})$ forms an equilibrium.*

4. *If $v \in [\max\{m_1(\epsilon), m_2\}, \min\{M_1, M_2\})$, the set of strategies $(t^{HE}, t^{LE}, o^{HNE}, u^{LNE})$ forms an equilibrium.*

5. *If $v \in [M_2, m_1(\epsilon))$, the set of strategies $(t^{HE}, t^{LE}, x^{HNE}, t^{LNE})$ forms an equilibrium.*

6. *If $v \in [\max\{m_1(\epsilon), M_2\}, M_1)$, the set of strategies $(t^{HE}, t^{LE}, o^{HNE}, t^{LNE})$ forms an equilibrium.*

7. *If $v \in [M_1, m_2)$, the set of strategies $(t^{HE}, t^{LE}, t^{HNE}, x^{LNE})$ forms an equilibrium.*

8. *If $v \in [\max\{m_2, M_1\}, M_2)$, the set of strategies $(t^{HE}, t^{LE}, t^{HNE}, u^{LNE})$ forms an equilibrium.*

9. *If $v \in [\max\{M_1, M_2\}, +\infty)$, the set of strategies $(t^{HE}, t^{LE}, t^{HNE}, t^{LNE})$ forms an equilibrium.*

Proof. We will prove the first four equilibria, i.e. when $v < M_1$ and $v < M_2$, which are the most general. The rest of the cases are similar to the proof of the Lemma 2.

We have that $v < \min\{M_1, M_2\}$. This means that $cq + v < c$ and $v < cq$. Consider the following general set of strategies:

- High Expert: If $C = 0$, he bids v. If $C = c$, he bids $c + v$.

- Low Expert: If $C = 0$, he does nothing. If $C = c$, he bids c.

- High nonexpert: He bids $cq + v$ with probability a and $c + \epsilon$ with probability $1 - a$.

- Low nonexpert: He bids v with probability g and cq with probability $1 - g$.

The probabilities a, g are as yet undetermined. We will examine if anyone has incentive to change strategy and at the same time try to determine the probabilities and the conditions for which the above is an equilibrium. These conditions will give us the proof that equilibria 1 and 3 are correct.

Both the high and the low experts bid truthfully and this is optimal for them. This is because they bid in the second stage, therefore they don't have any fear to reveal the common value to nonexperts. They also know their true valuation, and since we have a second-price auction, it is optimal for them to bid their true values.

Now, we consider a high nonexpert. His expected valuation is $cq+v$. Their payoff by bidding $cq+v$ is

$$\begin{aligned} A_2 = & \underbrace{prq(0)}_{\text{opponent is high expert and } C=c} \\ & + \underbrace{pr(1-q)(0)}_{\text{opponent is high expert and } C=0} \\ & + \underbrace{p(1-r)q(0)}_{\text{opponent is low expert and } C=c} \\ & + \underbrace{p(1-r)(1-q)(v)}_{\text{opponent is low expert and } C=0} \\ & + \underbrace{(1-p)r(0)}_{\text{opponent is high nonexpert}} \\ & + \underbrace{(1-p)(1-r)(g(cq)+(1-g)(v))}_{\text{opponent is low nonexpert}}, \end{aligned}$$

while their payoff by bidding $c+\epsilon$ is

$$\begin{aligned} B_2 = & \underbrace{prq(0)}_{\text{opponent is high expert and } C=c} \\ & + \underbrace{pr(1-q)(0)}_{\text{opponent is high expert and } C=0} \\ & + \underbrace{p(1-r)q(v)}_{\text{opponent is low expert and } C=c} \\ & + \underbrace{p(1-r)(1-q)(v)}_{\text{opponent is low expert and } C=0} \\ & + \underbrace{(1-p)r\left((1-a)\frac{cq+v-(c+\epsilon)}{2}\right)}_{\text{opponent is high nonexpert}} \\ & + \underbrace{(1-p)(1-r)(g(cq)+(1-g)(v))}_{\text{opponent is low nonexpert}}. \end{aligned}$$

By bidding $c+\epsilon+\zeta$ for some small $\zeta>0$, his payoff can only decrease. By bidding c, the payoff is the same as with the bid of $cq+v$. It holds that

$$B_2 - A_2 = p(1-r)q(v) + (1-p)r\left((1-a)\frac{cq+v-(c+\epsilon)}{2}\right),$$

and we want this to be equal to 0 to permit mixing these strategies, which will give us an expression for the mixing probability a. This is

$$a = 1 - \frac{2p(1-r)qv}{(1-p)r(c+\epsilon-(cq+v))}.$$

This is always ≤ 1. The inequality $a>0$ is equivalent to $v < \frac{(c+\epsilon-cq)(1-p)r}{2p(1-r)q+(1-p)r} = m_1$. So, we need this condition for equilibria 1 and 3. If $1-\frac{2p(1-r)qv}{(1-p)r(c+\epsilon-(cq+v))} \leq 0$, then it is always better for the high nonexpert to bid $c+\epsilon$, so we set $a=0$ (equilibria 2 and 4).

Next, we consider a low nonexpert. His expected valuation is cq. His payoff if he bids cq is

$$\begin{aligned} A_3 = & \underbrace{pr(q0 + (1-q)(-v))}_{\text{opponent is high expert}} \\ & + \underbrace{p(1-r)(q0 + (1-q)0)}_{\text{opponent is low expert}} \\ & + \underbrace{(1-p)r(0)}_{\text{opponent is high nonexpert}} \\ & + \underbrace{(1-p)(1-r)(g(cq-v) + (1-g)0)}_{\text{opponent is low nonexpert}}. \end{aligned}$$

His payoff if he bids v is

$$\begin{aligned} B_3 = & \underbrace{pr(q0 + (1-q)0)}_{\text{opponent is high expert}} \\ & + \underbrace{p(1-r)(q0 + (1-q)0)}_{\text{opponent is low expert}} \\ & + \underbrace{(1-p)r(0)}_{\text{opponent is high nonexpert}} \\ & + \underbrace{(1-p)(1-r)\left(g\frac{cq-v}{2}\right)}_{\text{opponent is low nonexpert}}. \end{aligned}$$

It must hold that $A_3 = B_3$ to permit mixing these strategies, from which we get an expression for the mixing probability g which is

$$g = \frac{2pr(1-q)v}{(1-p)(1-r)(cq-v)}.$$

This expression is always non-negative, and it is < 1 iff

$$v < \frac{c(1-p)(1-r)q}{2pr(1-q) + (1-p)(1-r)} = m_2.$$

If $v \geq m_2$, then we set $g = 1$ (which corresponds to strategy u^{LNE}).

This ends the proof. □

Notice that as ϵ goes to 0, the bidding strategies of Lemma 5 approach the strategies of the main Lemma 2. This means that the analogues of Proposition 1 and Proposition 2 will continue to hold with the alternative tie-breaking rule.

B.4 Distribution of Bidders' Private Value

In the main model, we assumed that V has binary distribution with support $\{0, v\}$. In this section we relax that assumption and show that our main result, that nonexperts sometimes bid more than their expected value, still holds. More specifically, for $k \geq 2$, we assume that the private value of each bidder is $V = \frac{i \cdot v}{k-1}$ with probability r_i, where $i \in \{0, 1, \ldots, k-1\}$ and $\sum_{i=0}^{k-1} r_i = 1$. In the main model, we had $k = 2$, $r_2 = r$, and $r_1 = 1 - r$.

The tie-breaking rule is a generalization of what we had in the main model. We assume that in case of a tie the bidder with the highest private value wins. If the two bidders have the same private value, then the nonexpert wins. If both bidders have the same private value and the same expertise level, then the winner is determined with a fair coin toss.

To simplify the analysis, we assume that $\delta = 0$ and only focus on the bids in the second stage. It is easy to see that it is weakly dominant for all the experts to bid their true valuation. In other words, if an expert has private value $\frac{i \cdot v}{k-1}$, he will bid $\frac{i \cdot v}{k-1}$ when the common value is low ($C = 0$), and $c + \frac{i \cdot v}{k-1}$, when the common value is high ($C = c$). We also assume that $v \leq c \cdot \min\{q, 1-q\}$ (which corresponds to the condition $v \leq \min\{M_1, M_2\}$ of our main model).

We show that, in equilibrium, nonexperts will mix between at most three different bids. More specifically, if a nonexpert is of type i, meaning that his private value is $\frac{i \cdot v}{k-1}$, he will mix between $\frac{(i+1) \cdot v}{k-1}$, $c \cdot q + \frac{i \cdot v}{k-1}$, and $c + \frac{(i-1) \cdot v}{k-1}$. The bid $\frac{(i-1) \cdot v}{k-1}$ is employed because he wants to lose against the experts of higher type when $C = 0$. The bid $c \cdot q + \frac{i \cdot v}{k-1}$ is used because this is his expected valuation and this is the bid he wants to have against a nonexpert. The bid $c + \frac{(i-1) \cdot v}{k-1}$ is used because he wants to win against an expert of lower type when $C = c$.

The reason that bidders use only these three bids in equilibrium is that, assuming that the opponent also uses the same strategy in equilibrium, every other bid is dominated by at least one of these three. The intuition is as follows. A bid $y \in [0, cq)$ will lose against all the bids in $[cq, +\infty)$ and will win only against some bids in $[0, v]$. But a bid in $[0, v]$ means that either the opponent is an expert and $C = 0$, in which case we want to win against bids in $[0, v_i]$ and lose against bids in $[v_{i+1}, v]$, or the opponent is a nonexpert who happened to underbid, in which case we want to win all the time, something achieved by the bid of $cq + v_i$. So, depending on the parameters of the model, y is dominated by either v_{i+1} or $cq + v_i$. When these two give the same payoff, i.e. when the nonexpert is mixing between the two, y is dominated by both.

Similarly, a bid $y \in [cq, c)$ will lose against all the bids in $[c, +\infty)$, will win against all bids in $[0, v]$, and will win against some bids in $[cq, cq+v]$. But a bid in $[cq, cq+v]$ means that the opponent is a nonexpert, in which case we want to win against all bids in $[cq, cq + v_{i-1}]$ and lose against bids in $[cq + v_{i+1}, cq + v]$. So, y is dominated by the bid $cq + v_i$.

Finally, a bid $y \in [c, +\infty)$ will win against all bids in $[0, cq + v]$, and will win against some bids in $[c, +\infty)$. But a bid in $[c, +\infty)$ means that either the opponent is an expert and $C = c$, in which case we want to win against bids in $[c, c + v_{i-1}]$ and lose against bids in $[c + v_i, v]$, or the opponent is a nonexpert who happened to overbid, in which case we want to lose all the time, something achieved by the bid of $cq + v_i$. So, depending on the parameters of the model, y is dominated by either $c + v_{i-1}$ or $cq + v_i$. When these two give the same payoff, i.e. when the nonexpert is mixing between the two, y is dominated by both.

Suppose that the nonexperts of type i will bid $\frac{(i+1) \cdot v}{k-1}$ with probability $\theta_{i,1}$, $c \cdot q + \frac{i \cdot v}{k-1}$ with probability $\theta_{i,2}$, and $c + \frac{(i-1) \cdot v}{k-1}$ with probability $\theta_{i,3}$, where $\theta_{i,1} + \theta_{i,2} + \theta_{i,3} = 1$. It holds that $\theta_{0,3} = 0$ and $\theta_{k-1,1} = 0$.

The expected payoff of a nonexpert of type $i < k-1$ when he bids $\frac{(i+1) \cdot v}{k-1}$ is

$$\Phi_i = (1-p) \left(\sum_{j=0}^{i-1} r_j \theta_{j,1} \left(cq + \frac{iv}{k-1} - \frac{(j+1)v}{k-1} \right) + \frac{1}{2} r_i \theta_{i,1} \left(cq + \frac{iv}{k-1} - \frac{(i+1)v}{k-1} \right) \right)$$

$$+ p(1-q)\sum_{j=0}^{i}\frac{v(i-j)r_j}{k-1}.$$

The expected payoff of a nonexpert of type i when he bids $c \cdot q + \frac{i \cdot v}{k-1}$ is

$$\begin{aligned}\Psi_i =&(1-p)\left(\sum_{j=0}^{k-1} r_j\theta_{j,1}\left(cq+\frac{iv}{k-1}-\frac{(j+1)v}{k-1}\right)+\sum_{j=0}^{i-1} r_j\theta_{j,2}\left(\frac{iv}{k-1}-\frac{jv}{k-1}\right)\right)\\ &+ p(1-q)\sum_{j=0}^{k-1}\frac{v(i-j)r_j}{k-1}.\end{aligned}$$

The expected payoff of a nonexpert of type $i > 0$ when he bids $c + \frac{(i-1)\cdot v}{k-1}$ is

$$\begin{aligned}\Omega_i =&(1-p)\left(\sum_{j=0}^{k-1} r_j\theta_{j,1}\left(cq+\frac{iv}{k-1}-\frac{(j+1)v}{k-1}\right)+\sum_{j=0}^{i-1} r_j\theta_{j,3}\left(cq-c+\frac{iv}{k-1}-\frac{(j-1)v}{k-1}\right)\right.\\ &\left.+\sum_{j=0}^{k-1} r_j\theta_{j,2}\left(\frac{iv}{k-1}-\frac{jv}{k-1}\right)+\frac{1}{2}r_i\theta_{i,3}\left(cq-c+\frac{iv}{k-1}-\frac{(i-1)v}{k-1}\right)\right)\\ &+p\left((1-q)\sum_{j=0}^{k-1}\frac{v(i-j)r_j}{k-1}+q\sum_{j=0}^{i-1}r_j\left(\frac{iv}{k-1}-\frac{jv}{k-1}\right)\right).\end{aligned}$$

Consider the case where $\theta_{i,1} > 0$ for every $i < k-1$, $\theta_{i,2} > 0$ for every i, and $\theta_{i,3} > 0$ for every $i > 0$. In other words, all types of nonexperts are mixing between all their potential bids. This case corresponds to the equilibrium in case 1 of Lemma 2. To find all the probabilities t, we need to solve the system

$$\begin{aligned}&\Phi_i = \Psi_i, \quad \text{for } i \in \{0,1,\ldots,k-2\}\\ &\Psi_i = \Omega_i, \quad \text{for } i \in \{1,2,\ldots,k-1\}\\ &\theta_{0,3} = 0\\ &\theta_{k-1,1} = 0\\ &\sum_{m=1}^{3}\theta_{i,m} = 1, \quad \text{for } i \in \{0,1,\ldots,k-1\}.\end{aligned}$$

The solution to this system for $k = 2$ is

$$\begin{aligned}\theta_{0,1} &= \frac{2p(q-1)r_1v}{(p-1)r_0(cq-v)}\\ \theta_{0,2} &= 1-\frac{2p(q-1)r_1v}{(p-1)r_0(cq-v)}\\ \theta_{0,3} &= 0\\ \theta_{1,1} &= 0\\ \theta_{1,2} &= 1-\frac{2pqr_0v}{(p-1)r_1(c(q-1)+v)}\\ \theta_{1,3} &= \frac{2pqr_0v}{(p-1)r_1(c(q-1)+v)},\end{aligned}$$

which is the same as our solution in the main body of the paper ($\theta_{1,2} = a$ and $\theta_{0,1} = g$).

The solution to the system for $k = 3$ is

$$\theta_{0,1} = \frac{2v(2cq(p(q-1)r_1 + 2(p-1)r_0) + v(-p(q-1)(r_1 - 2r_2) - 4(p-1)r_0))}{(p-1)r_0\left(4c^2q^2 + 4cqv - 7v^2\right)}$$

$$\theta_{0,2} = \frac{(2cq - v)((p-1)r_0(2cq - v) - 2p(q-1)r_1v) - 4p(q-1)r_2v^2}{(p-1)r_0\left(4c^2q^2 + 4cqv - 7v^2\right)}$$

$$\theta_{0,3} = 0$$

$$\theta_{1,1} = \frac{2v(2cq(p(q-1)r_2 - (p-1)r_0) + v(p(q-1)(2r_1 + 3r_2) + (p-1)r_0))}{(p-1)r_1\left(4c^2q^2 + 4cqv - 7v^2\right)}$$

$$\begin{aligned}\theta_{1,2} = \Big(&16c^4(p-1)(q-1)^2q^2r_1 - 16c^3(q-1)qv\Big(r_2(pq^2 - (p+1)q + p)\\
&+ r_0(p((q-1)q+1) + q - 1) + (p-1)r_1\Big)\\
&+ 4c^2v^2\Big(2r_0(p(q((q-1)q+4)-1) + 3q^2 - 4q + 1)\\
&+ r_1(p(-6(q-1)q-3) + 18(q-1)q + 7) - 2r_2(p(q((q-2)q+5)-3) + q(2-3q))\Big)\\
&+ 4cv^3\Big(r_0(p(q(13q-12)-2) + 5q + 2) + r_1(p(8(q-1)q-3) + 7)\\
&+ r_2(p(q(13q-14)-1) - 5q + 7)\Big) + 7v^4\Big(2r_0(-3pq + p - 1)\\
&+ 2r_2(3pq - 2p - 1) + (3p-7)r_1\Big)\Big)\\
&\Big/\Big((p-1)r_1\left(4c^2(q-1)^2 - 4c(q-1)v - 7v^2\right)\left(4c^2q^2 + 4cqv - 7v^2\right)\Big)\end{aligned}$$

$$\theta_{1,3} = \frac{2(p-1)r_2v(2c(q-1)+v) - 2pqv((3r_0 + 2r_1)v - 2c(q-1)r_0)}{(p-1)r_1\left(4c^2(q-1)^2 - 4c(q-1)v - 7v^2\right)}$$

$$\theta_{2,1} = 0$$

$$\theta_{2,2} = \frac{(p-1)r_2(2c(q-1)+v)^2 + 2pqv(2r_0v - r_1(2c(q-1)+v))}{(p-1)r_2\left(4c^2(q-1)^2 - 4c(q-1)v - 7v^2\right)}$$

$$\theta_{2,3} = \frac{2pqv(r_1(2c(q-1)+v) - 2r_0v) - 8(p-1)r_2v(c(q-1)+v)}{(p-1)r_2\left(4c^2(q-1)^2 - 4c(q-1)v - 7v^2\right)}.$$

Even though we can analytically solve the system for larger values of k, the closed-form solution does not have any meaningful pattern. In the following example, we numerically solve the system for the case of $k = 20$, which would be an approximation of uniform continuous distribution of v.

Figure 10 shows a plot of the three mixing probabilities as functions of the private value of a nonexpert for $k = 20$ and small v. We can see that as the private value increases, the probability of underbidding decreases and the probability of overbidding increases.

As v increases, we will get different equilibria where some types of nonexperts don't mix between all their potential bids, i.e., some $\theta_{i,1}$'s become 0. A complete analysis of the equilibrium is beyond the scope of this paper as the number of cases in equilibrium analysis grows exponentially in k as v increases; however, the general pattern is that as v increases,

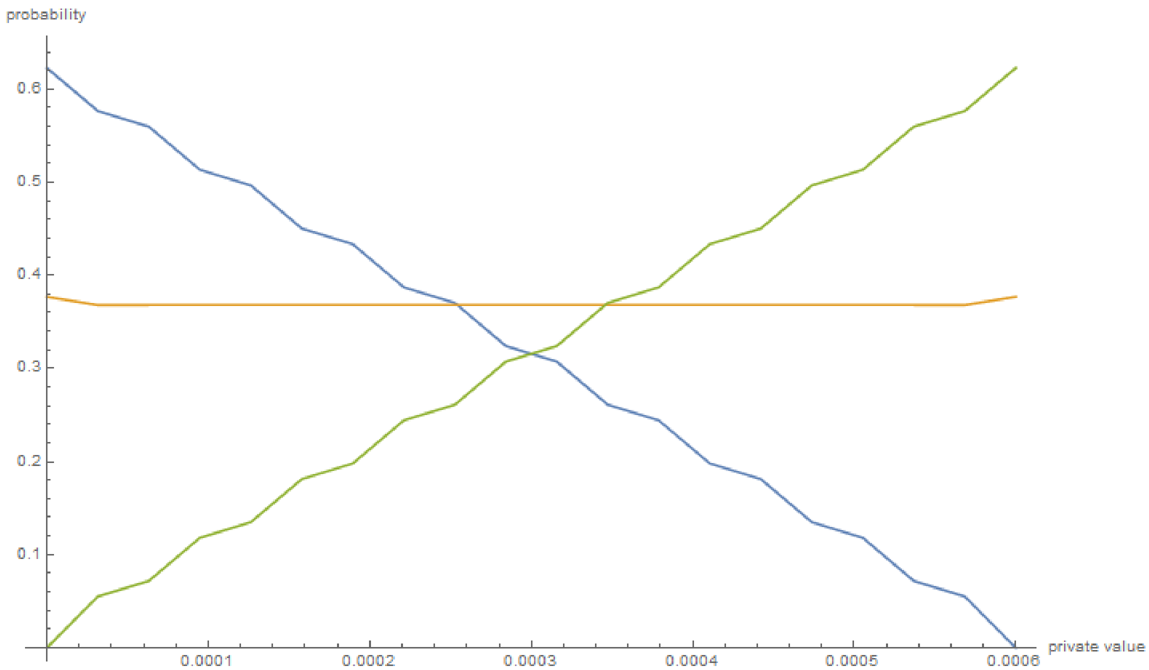


Figure 10: Mixing probabilities as a function of the private value. Blue is for underbidding, orange for bidding the expected valuation, and green for overbidding. The plot is for $k = 20$, $r_i = 1/k$ for $i \in \{0, \ldots, k-1\}$, $c = 1$, $q = 0.5$, $p = 0.999$, and $v = 0.0006$.

nonexperts bid more aggressively. This is consistent with our findings in the main body of the paper.

B.5 Signaling Using Closing Format: Hard vs. Soft Close

In this section, we consider a situation in which the platform lets the sellers decide whether to sell in an auction with hard-close or soft-close format. We call the seller of an item with high common value a high-type seller, and the seller of an item with low common value a low-type seller. A seller is high-type with probability q where q is common knowledge. A seller naturally knows his own type; experts also know the seller's type (since they know the common value of items being offered). But nonexperts do not know the seller's type. We investigate whether a seller can signal his type using the closing format (soft versus hard). In particular, we derive conditions for existence of a separating equilibrium.

A seller sets his closing format F (soft or hard). For a format F, we assume that all nonexperts have the same belief about a seller who uses F. In general, nonexperts' belief about a format is the probability that they think a seller using that format is high-type. However, since we only consider pure strategy Nash equilibria of the game, nonexperts' belief about a format is limited to three possibilities: Low (L), High (H), and Unknown (X). In belief L, nonexperts believe that a seller using format F is always a low-type seller. In belief H, nonexperts believe that a seller using format F is always a high-type seller. Finally, in belief X, nonexperts cannot infer anything about the seller's type and believe that the seller is high-type with probability q.

Nonexperts have beliefs about each format F. In equilibrium, the beliefs must be consistent with sellers' strategies. In particular, if both types of sellers use the same format in (a pooling) equilibrium, nonexperts' belief for that format must be X. If the two types of sellers use different formats in (a separating) equilibrium, nonexperts' belief for the format used by the low-type seller must be L and for the format used by the high-type seller must be H. Furthermore, in an equilibrium, given the nonexperts' beliefs, sellers should not be able to benefit from changing their strategies.

We use the following notation to explain the results of this section: Let $\pi_T^B(F)$, where $T \in \{L, H\}$ and $B \in \{L, H, X\}$ denote the expected profit of a seller who uses mechanism $F \in \{soft, hard\}$ and has type T, and nonexperts believe has type B.

Lemma 6. *For any $B \in \{L, H\}$ and any $T \in \{L, H\}$ we have $\pi_T^B(soft) = \pi_T^B(hard)$. In other words, if nonexperts have no uncertainty about the type of the seller ($B \neq X$), soft-close and hard-close formats both lead to the same revenue for the seller (no matter what type the seller is).*

Proof. Note that when nonexperts have no uncertainty about the type of the seller, they do not infer anything from other bidders' bids, and do not update their expected value. The auction reduces to a full-information second price auction in this case. □

The seller's revenue, in each case, is given by

$$\pi_H^H(soft) = \pi_H^H(hard) = c + r^2 v;$$

$$\pi_L^L(soft) = \pi_L^L(hard) = r^2 v;$$

$$\pi_H^L(soft) = \pi_H^L(hard) = \begin{cases} cp^2 + rv(2(1-p)p(1-r) + r) & \text{if } v \leq c, \\ cp\left(2(1-p)r + p - 2(1-p)r^2\right) + r^2 v & \text{if } v > c; \end{cases}$$

$$\pi_L^H(soft) = \pi_L^H(hard) = \begin{cases} c(1-p)^2 + rv(2(1-p)p(1-r) + r) & \text{if } v \leq c, \\ c(1-p)(p(2(1-r)r - 1) + 1) + r^2 v & \text{if } v > c. \end{cases}$$

Lemma 7. *No separating equilibrium exists.*

Proof. Assume for sake of contradiction that there is a separating equilibrium in which the low-type uses format F and the high-type uses format F'. Note that, using the above expressions, we have $\pi_L^L(F) < \pi_L^H(F) = \pi_L^H(F')$ for any F and F'. Therefore, the low-type benefits from mimicking, contradicting the equilibrium condition. □

Depending on out-of-equilibrium beliefs, the game could have multiple pooling equilibria (both soft- and hard-close). We can show that soft-close is always a pooling equilibrium. Furthermore, for regions in which hard-close provides higher expected revenue for the high-type seller, as shown in Figure 6, hard-close is also a pooling equilibrium. The intuitive criterion is not sufficient for refining the equilibrium set to a unique equilibrium. However, we can show that only soft-close pooling equilibrium can survive the D1 criterion refinement.

Intuitively, the high-type always gains more (loses less) than low-type by deviating to soft-close format in a hard-close pooling equilibrium. Therefore, out-of-equilibrium beliefs on soft-close auction, subject to D1 requirement, is high. This makes the deviation to soft-close always profitable (in a hypothetical hard-close equilibrium). Therefore, hard-close pooling equilibrium cannot survive D1 criterion refinement. This is formally proved in the following lemma.

Lemma 8. *A hard-close pooling equilibrium cannot survive D1 criterion refinement.*

Proof. Assume for sake of contradiction that there is a hard-close pooling equilibrium. Let z be the buyer's belief, the probability that the seller is high-type, on observing the soft-close format. We show that for any $z < 1$, if a low-type seller weakly benefits from deviating to soft-close, a high-type seller strictly benefits from deviating. Then, according to D1 criterion, this implies that out-of-equilibrium belief on soft-close has to be high. Therefore, hard-close cannot be an equilibrium.

Assume for sake of contradiction that there is a z for which a low-type seller weakly benefits from deviating to soft-close, but a high-type seller does not strictly benefit from deviating to soft-close. First, note that if both buyers are nonexperts, the equilibrium outcome is not affected by the type of the seller. In other words, both types of sellers would have the same revenue in each closing format. Similarly, if both buyers are experts, the equilibrium outcome is not affected by the closing format. Therefore, to compare the benefit of deviation (for sellers), we can assume that the buyers have different levels of expertise: an expert and a nonexpert.

If the expert is low-value, then the revenue is zero in both hard-close and soft-close formats for the low-type seller. The revenue for the high-type seller is always greater than or equal in soft-close (always c) than in hard-close (at most c, depending on the value of z and whether the nonexpert is using an aggressive strategy or not).

Finally, consider the case that the expert is high-value. First, assume that the nonexpert is also high-value. In this case, the revenue of soft-close for high-type seller is always c and for low-type seller is always v. The revenue of hard-close for the low-type seller is always v. Therefore, the low-type cannot prefer hard-close to soft-close in this sub-case. Next, assume that the nonexpert is low-value. In this case, the revenue of the auction in both soft-close and hard-close cases is determined by the bid of the nonexpert. A low-value nonexpert has the exact same strategy in soft-close and hard-close formats. Furthermore, this strategy does not depend on whether the common value is high or low. Therefore, low-type and high-type sellers have the same revenue in the hard-close format and in the soft-close format. As shown, there is no case (for any z) in which a low-type seller benefits from deviating to soft-close while a high-type seller does not. Furthermore, it is easy to see that there are cases in which the high-type seller strictly benefits from this deviation. Therefore, according to D1 criterion, buyers' out-of-equilibrium belief on soft-close auction has to be high.

Given that, in a hypothetical hard-close equilibrium, buyers' belief on soft-close is high, sellers always benefit from deviating to soft-close. Therefore, hard-close cannot be a pooling equilibrium. □

Finally, note that a soft-close pooling equilibrium always survives D1 criterion refinement. This is because whenever the high-type seller benefits from deviating to hard-close, the low-type seller also benefits from deviating to hard-close (the proof is very similar to the proof of Lemma 8). Therefore, a soft-close pooling equilibrium in which out-of-equilibrium belief on hard-close is low survives D1 criterion refinement.

Given that the only pure strategy equilibrium that survives D1 criterion refinement is a soft-close pooling equilibrium, we show that, compared to the case where the platform decides closing format, a low-type seller and the platform are both (weakly) worse off if the closing format decision is left to the sellers. A high-type seller may be worse off or better off, depending on other parameters, as shown in Figure 6.