The Beneficial Effects of Ad Blockers

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Abstract

Although online advertising is the lifeline of many internet content platforms, the usage of ad blockers has surged in recent years, presenting a challenge to platforms dependent on ad revenue. Using a simple analytical model with two competing platforms, we show that the presence of ad blockers can actually benefit platforms. In particular, there are conditions under which the optimal equilibrium strategy for the platforms is to allow the use of ad blockers (rather than using an adblock wall, or charging a fee for viewing ad-free content). The key insight is that allowing ad blockers serves to differentiate platform users based on their disutility to viewing ads. This allows platforms to increase their ad intensity on those that do not use the ad blockers and achieve higher returns than in a world without ad blockers. We show robustness of these results when we allow a larger combination of platform strategies, as well as by explaining how ad whitelisting schemes offered by modern ad blockers can add value. Our study provides general guidelines for what strategy a platform should follow based on the heterogeneity in the ad sensitivity of their user base.

Keywords: ad blocking; ad sensitivity; ad intensity; competitive strategy; advertising; game theory

1 Introduction

1.1 Background

Online ads are like taxes. Nobody likes them, but they exist because people understand that they are necessary. Millions of websites, including some of the largest internet companies, depend on advertising as their main source of revenue. Online advertising revenue in the US in 2015 was \$59.6 billion¹, almost half of it accounted for by Google.^{2,3} Google, Facebook⁴, and Twitter⁵ together make up more than 65% of the total revenue.⁶ Advertising is the main source of revenue for all these companies. Another example of an industry that depends heavily on advertising is the US news industry, with 69% of its revenue coming from advertising.⁷ More generally, advertising is the key reason many content-providing websites are able to offer their services to users for "free" (other than the implicit payment of user attention to the ads). In short, today's internet would not be what it is without advertising.

Of course advertising is not a new phenomenon. Even before the era of the internet, companies advertised products on billboards, newspapers, radio stations, TV channels, and other mass media. However, there is a key difference between advertising on the internet today and the other media. The interactive nature of the internet gives users the easy ability to block ads with ad blockers. An ad blocker is a type of software, usually added conveniently as an extension to an internet browser, that will prevent any ads from appearing on the browsed web pages. When a user with an ad blocker visits a website with ads, the blocker identifies the ad content and blocks it from loading. Consequently, the website does not receive any ad revenue for that user.⁸

Ad blocking is not something new either. After VCRs became popular in the 1980s, there was a trend among viewers for commercial skipping. To combat this, advertisers tried to make ads more entertaining. In 1999, ReplayTV launched the first DVR with a built-in feature to skip commercials.⁹ Since then, providers of commercial skipping features have been plagued by lawsuits that claim damages to the copyright of the original content.¹⁰ A difference between these precursors and ad blockers on the internet is that now it is easier than ever before to block ads, since several ad-blocking extensions are just a few clicks away

¹http://www.iab.com/wp-content/uploads/2016/04/IAB-Internet-Advertising-Revenue-Report-FY-2015.pdf (accessed October 19, 2019)

 $^{^{2} \}rm https://www.statista.com/statistics/266249/advertising-revenue-of-google/ (accessed October 19, 2019)$

³http://www.businessinsider.com/stats-on-googles-revenues-from-youtube-and-googleplay-2015-7 (accessed October 19, 2019)

⁴http://www.businessinsider.com/facebook-us-ad-revenue-2016-1 (accessed October 19, 2019)

⁵https://www.emarketer.com/Article/Twitter-Ad-Revenues-Expected-Continue-Robust-Growth/1013571 (accessed October 19, 2019)

⁶https://www.bloomberg.com/news/articles/2016-04-22/google-and-facebook-lead-digitalad-industry-to-revenue-record (accessed October 19, 2019)

⁷http://www.journalism.org/2014/03/26/the-revenue-picture-for-american-journalism-andhow-it-is-changing/ (accessed October 19, 2019)

⁸See e.g., http://alternativeto.net/software/adblock-plus/ (accessed October 19, 2019).

⁹https://en.wikipedia.org/wiki/Commercial_skipping (accessed October 19, 2019)

¹⁰https://www.techdirt.com/articles/20160211/10423633579/dish-agrees-to-cripple-adskipping-dvr-to-settle-fox-lawsuit.shtml (accessed October 19, 2019)

in most browsers.¹¹



Figure 1: Millions of devices with adblock software over the years (— Desktop, - - Mobile). (PageFair, 2017)

In Figure 1, we can see how adblock usage is changing over the years for desktop and mobile devices. We observe a steady increase in both categories with an average of 44.8% increase for desktop and 63.9% increase for mobile per year. Even though mobile ad blockers were not as popular as their desktop counterparts in 2015, in the beginning of 2017 we see the opposite, since more than 380 million mobile devices have an installed ad blocker versus 236 million desktop devices. PageFair and Adobe (2015) estimated that the cost of ad blockers for publishers in terms of lost revenue in 2015 was \$21.8 billion, which was around 14% of the global ad spend. Today with many more devices with ad blockers than in 2015 (Figure 1), we expect this number to be much larger.

1.2 How do Platforms Respond?

Websites hosting content and supported by ads act as platforms for gathering viewers and advertisers. Their revenue stream is directly affected by the deployment of ad blockers by the viewers. The response of these platforms to ad blockers have varied considerably.^{12,13}

Some platforms disallow the use of ad-blocking software when viewing their sites, by using an **adblock wall**. This is the name for currently available technology that allows websites to detect if a visitor is using an ad blocker and if so, refuse to give access to him. Forbes is an example of a website that uses an adblock wall.¹⁴ City A.M. was the first UK newspaper website to ban the use of ad blockers and prevent adblock users from reading content.¹⁵

¹¹https://adblockplus.org/getting_started (accessed October 19, 2019)

¹²https://www.pubnation.com/blog/publishers-fight-back-how-the-top-50-websites-combatadblock (accessed May 2017)

¹³http://adage.com/article/media/publishers-weigh-ways-fight-ad-blocking/299116/ (accessed October 19, 2019)

¹⁴https://digiday.com/media/forbes-ad-blocking/ (accessed October 19, 2019)

¹⁵https://www.theguardian.com/media/2015/oct/20/city-am-ban-ad-blocker-users (accessed October 19, 2019)

Other platforms offer ad-free or ad-light subscription services for viewing content, by using a **paywall**. Financial Times¹⁶, The Wall Street Journal¹⁷, and The Washington Post¹⁸ are a few examples of news sites with such a paywall. A slightly different but related strategy was adopted by YouTube: it was originally dependent solely on advertising, but in 2015 it launched YouTube Red, a subscription based service that offers ad-free access to all YouTube videos with some additional exclusive content.¹⁹

Many platforms use a combination of the aforementioned options. They use an adblock wall that offers two choices to the users, either to disable their ad blocker or pay a fee for an ad-free version of the site. Some examples of websites using this strategy are Wired²⁰, Bild²¹, and Business Insider.²² The New York Times has also experimented with an adblock wall of this type for some time.²³ This option mirrors ad-free services that have been available in more traditional media, e.g. an alternate to watching a movie or show for free on network TV is to buy or rent an ad-free copy.

Finally, there are some platforms, like The Guardian, that request viewers to disable ad blockers as a gesture of support for the content in the site (without preventing access if they do not).²⁴ There are also sites that simply ignore the use of ad blockers and **allow** their use. In fact, the majority of the content providing websites in the internet today follow this simple strategy of doing nothing about the existence of ad blockers other than simply allowing their use.

1.3 Research Questions

Internet ad blockers motivate some fundamental questions: What is the optimal response of platforms to their presence? Are adblock walls the solution to the ad-blocker problem? Why should platforms ever allow ad blockers, if they can prevent them using a simple adblock wall? When should they erect a paywall and charge a fee for ad-free or ad-light content? Under what conditions should they use these different options? In this paper, we address the central questions above, and explore further the effects ad blockers have on platforms and users. For instance, if we compare a world without ad blockers and the current world with them, are the effects of ad blocking only negative for platforms? How do platforms' ad-revenues change with the availability of ad blockers? How is overall user welfare affected

¹⁶https://www.ft.com/products (accessed October 19, 2019)

¹⁷https://subscribe.wsj.com (accessed October 19, 2019)

¹⁸https://subscribe.washingtonpost.com (accessed October 19, 2019)

¹⁹https://www.theverge.com/2015/10/21/9566973/youtube-red-ad-free-offline-paidsubscription-service (accessed October 19, 2019)

²⁰https://www.wired.com/how-wired-is-going-to-handle-ad-blocking/ (accessed October 19, 2019)

²¹http://www.reuters.com/article/us-axelspringer-adblock-idUSKCN0S70S020151013 (accessed October 19, 2019)

²²http://adage.com/article/media/business-insider-testing-paywall-ad-blockingresponse/305951/ (accessed October 19, 2019)

²³https://www.wsj.com/articles/new-york-times-experiments-with-ways-to-fight-adblocking-1457378218 (accessed October 19, 2019)

²⁴http://www.economist.com/news/business/21653644-internet-users-are-increasinglyblocking-ads-including-their-mobiles-block-shock (accessed October 19, 2019)

by ad blocking? To answer these questions, there are three important elements we model: competition, ad intensity, and heterogeneity in the ad sensitivity.

Competition. There are several reports that adblock walls do not work.²⁵ Several websites that implemented some type of adblock wall, like those of Wired, Bild, and Forbes, have seen their traffic deteriorate right after the introduction of the adblock wall.²⁶ The main explanation for this is that most adblock users who visit such websites and face a wall prefer to leave the website instead of disabling their ad blocker, even temporarily. In fact, in a survey by PageFair (2017), 74% of adblock users said that they leave websites when faced with an adblock wall, and only 26% disable their ad blockers to read the content.

Competition is the key reason for why adblock walls do not work. Most websites do not offer unique content that users cannot find elsewhere. As a result, instead of disabling their ad blocker and facing the inconvenience of ads, users prefer to look for the same or similar content elsewhere.

Ad intensity. Websites can control how many ads they will show, how intrusive or annoying the ads will be, their size, their position, and so on. All these affect the user experience and how much disutility a user will get from the ads. As an example, Forbes.com, when presenting an adblock wall to a user, shows a message promising users that if they disable their ad blocker, they will be presented with an "ad-light" experience in return. In the survey by PageFair (2017), 77% of adblock users said that they were willing to view some ad formats and are not totally against ads. Therefore, ad intensity is a key decision for platforms, since it directly affects how users react.

Heterogeneity in ad sensitivity. The increase in the adoption of ad blockers has other reasons in addition to their ease of installation. Digital ads offering rich-media content such as audio, video, pop-ups and flashing banners have become increasingly intrusive to the content absorption experience. The rise of the mobile internet also puts a premium on the space available for content viewing that is jeopardized by ads that take up too much realestate, mobile data consumption, and battery life. Finally, re-targeting practices associated with digital ads have increased the perception of privacy intrusion among viewers.

Nevertheless, the adoption of ad blockers among viewers is not likely to be universal since the sensitivity of viewers to ads is sufficiently heterogeneous across sites and devices from which the sites are accessed. Users that access the platforms from public or corporate machines may not have the ability to install new uncertified software such as ad blockers. Casual users who do not spend too much time on sites with annoying ads will not take the effort to employ ad blockers and update/maintain them. Less technical users may not even be aware of the existence and convenience of ad blockers. Many technically-savvy users may also continue to allow ads to support the sites they visit by acknowledging that they indirectly pay for the content they consume. Some users simply continue to view ads so as be kept informed of new products and promotions over time.

In a survey by PageFair and Adobe (2015), when non-adblock users were asked what would cause them to start using an ad blocker, 50% of the respondents stated that misuse of

²⁵http://www.businessinsider.com/ad-blocking-walls-not-working-2016-2 (accessed October 19, 2019)

²⁶https://techerati.com/the-stack-archive/world/2016/04/21/sites-that-block-adblockersseem-to-be-suffering/ (accessed October 19, 2019). However, it is unclear whether the loss in traffic after an adblock wall implementation directly translates to loss in revenue.

their personal information would be a reason to enable ad blocking. 41% of them responded that an increase in the number of ads from what they typically encounter today would also be a good reason. There was an 11% saying that they would never use an ad blocker, and this proportion increases to 23% for those aged between 35 and 49 years old. In contrast, when adblock users were asked for their main motivation behind adblock usage in PageFair (2017), only 6% of them stated privacy as the main reason. Security and interruption were the two leading reasons at 30% and 29% respectively, while page speed and the fact that there are too many ads came next with 16% and 14% respectively.

This provides evidence that there are fundamentally two classes of users based on whether they use ad blockers or not, and there is a lot of heterogeneity in the ad sensitivity of users in both classes. Furthermore, there is also difference in ad sensitivity between these two classes of adblock and non-adblock users.

1.4 Contributions

In this paper we devise a simple analytical model to answer the questions of Section 1.3. We model sites as two competing platforms for hosting content and attracting users. We assume two classes of users: one that uses ad blockers and the other that does not; note that the former are typically more ad-sensitive than the latter. Each platform has three options:

- BAN strategy: Continue displaying ads and ban ad blocking (e.g., using an adblock wall). If a viewer uses an ad blocker, he has to disable it to get access to the site.²⁷
- ALLOW strategy: Continue to display ads and allow ad-blocking software by any user that installs it.
- FEE strategy: Stop displaying ads and offer only an ad-free site with a subscription fee.

Note that in the second option, the platform will make no revenue from adblock users, but only from those who do not use an ad blocker and can see ads.

Given that banning ad blockers is an option for both platforms, we would expect that this would always emerge as an equilibrium strategy since it would curb the loss of revenues compared to the allow strategy. However the competitive dynamics between even two symmetric platforms results in a surprising equilibrium. Our first result argues that **there are conditions where both platforms arrive at ALLOW as their symmetric optimal strategies** (Proposition 1 in Section 4). The intuition is that the action of installing ad blockers serves as a filter for more ad-sensitive users that employ ad blockers; with these users gone, each platform can move to a higher intensity of advertising to users and hence increase revenue.

As allowing ad blockers results in increased advertising by both platforms, we may expect the utility of users exposed to this increased advertising to decrease substantially as a result.

 $^{^{27}}$ Another strategy some platforms use to bypass ad blockers is placing advertising content, e.g. mentions of products, that is organically mixed in with their native content. For the purposes of our model, this strategy can be considered the same as the BAN strategy, since it is just a different way to make adblock users see ads.

However, our second result argues that when platforms ALLOW ad blockers, this can increase the overall welfare of users (Proposition 2 in Section 4). The result follows from the filtering effect, which can raise the utility of ad-sensitive users substantially by allowing them to filter ad content, overshadowing the potential loss of utility to less ad-sensitive users who might now be subject to more advertising. Perhaps even more surprisingly though, there are cases where no user is worse off when ad blockers are allowed, while platforms and some users are better off resulting in a Pareto-improvement in overall welfare as a result of introducing ad blockers.

In Section 5.1, we extend the main model by adding the following option for platforms:

• ADS OR FEE strategy: Give the choice to users to either disable their ad blockers and be exposed to ads, or pay a fee for an ad-free version of the site (e.g., using a paywall).

The argument in favor of this new strategy is that it can achieve the filtering effect that the ALLOW strategy had by making ad-sensitive users pay the subscription fee, while users with lower ad sensitivity see ads. However, we show that **even with the addition of the ADS OR FEE strategy, there is still an equilibrium where both platforms ALLOW ad blockers** (Proposition 3). To show this, we further split the class of non-adblock users into two further classes with different ad sensitivities. In this context, ADS OR FEE is a better strategy for platforms when there is heterogeneity in the ad sensitivity between the two classes of non-adblock users, because it helps separate very ad-sensitive non-adblock user classes are more homogeneous in their ad sensitivity.

In Section 5.2, we extend the main model in a different direction by adding the following option for each platform:

• WHITELIST strategy: Allow ad blockers and pay a fee to the ad-blocker company to put the platform in the default whitelist.

This strategy is motivated by the "Acceptable Ads initiative"²⁸. This is a program introduced by Adblock Plus, the most popular adblock extension, according to which publishers and advertisers who comply with certain criteria could get whitelisted so that their ads may pass through the filter of the ad blocker. This strategy seems to be inferior to the previous strategies, since this option simply requires platforms to pay for something they had for free before the advent of ad blockers. However, we show that **there is an equilibrium where platforms use the WHITELIST strategy and this equilibrium can sometimes increase their revenue even more than when ad blockers did not exist** (Proposition 4). For this, we now split the class of adblock users into two further classes with different ad sensitivities. In this context, WHITELIST is a better strategy when there is heterogeneity in the ad sensitivity between the two classes of adblock users, since it can help platforms separate ad-sensitive adblock users from the rest, while ALLOW is a better strategy when the two adblock user classes are more homogeneous in their ad sensitivity.

Finally, in Section 6, we extend the main model to include content creators who generate the content of the platforms and share the revenue with them. We show the robustness of our earlier results in this extension; we also show that allowing ad blockers can result in an

²⁸https://adblockplus.org/acceptable-ads (accessed October 19, 2019)

increased quality of content. This provides an additional benefit for users when ad blockers are allowed.

2 Literature Review

Our paper is related to the advertising and marketing avoidance literature (Clancey, 1994; Cho and Cheon, 2004; Speck and Elliott, 1997; Li and Huang, 2016; Seyedghorban et al., 2016). Below we discuss some of the more closely related papers.

Anderson and Gans (2011) consider a model of a content provider who chooses a level of advertising while consumers decide if they will adopt ad-avoidance technology or not. They show that ad-avoidance penetration can increase advertising clutter, but it decreases the content provider's profit. One difference with our setting is that in their model, there is a price for consumers to adopt ad-avoidance technology.²⁹ As a result, their setting is more appropriate for more traditional ways of ad avoidance, like DVRs or other physical appliances, where there is a non-zero sunk cost for their adoption. In our setting, where the most popular ad blockers are free of charge and they are just a few clicks away to install in most browsers, this assumption is not as realistic. Another difference is that we consider a model with competition where platforms can actually decide if they will allow ad blocking or not. This is a more appropriate model for a website trying to decide if they will implement an adblock wall or not, instead of just assuming that adblock usage is unavoidable as in their model.

Johnson (2013) examines a model with firms that can target their ads to consumers and consumers who can avoid advertising. He shows that improved targeting can benefit firms but not necessarily consumers. He also shows, that in equilibrium, consumers may under-utilize their ability to block ads. A difference with our model is that there is a direct link between advertising firms and consumers, where firms can target the consumers with the higher probability of buying their product. In other words, there is no intermediate publisher who takes part in the decision process. He also assumes that there is some (positive) cost to consumers for avoiding ads and that the firm has a cost for sending an ad regardless of whether the ad is avoided or not. All these make his setting a better model for more traditional direct advertising campaigns, like direct mail with intentional avoidance by consumers.

Hann et al. (2008) take a different approach by focusing more on the privacy of consumers. In their setting, sellers market their products to consumers through solicitations. Consumers have two ways to avoid solicitations: either by concealment (e.g. registering in a do-not-call list) or by deflection (e.g. with call screening). There are also two types of consumers, consumers with high demand and those with low demand for the products. They show that concealment by low-demand consumers can lead sellers to market more, while the opposite is true when there is concealment by high-demand consumers. They also show that concealment is worse for consumer welfare than deflection.

²⁹In that sense, ad avoidance from the perspective of consumers is more like the ADS OR FEE plan of our model, but the content provider does not receive the fee. This can explain some of their results when viewed in our framework.

Wilbur (2008) studies a two-sided empirical model of the television industry with advertisers on one side and viewers on the other. One of his counterfactual findings is that ad avoidance tends to increase advertising quantities and decrease network revenues. Goh et al. (2015) investigate the externalities imposed by consumers who avoid ads on other consumers in the context of the US Do Not Call registry (DNC). They found that the number of subsequent DNC registrations was positively correlated with the number of first wave registrations. This suggests that perhaps telemarketers increased the number of calls to unregistered consumers after the first wave driving even more subsequent registrations.

Aseri et al. (2018) study a similar problem to ours, namely the benefits of ad blockers, in a different setting. They consider a monopolistic platform who might not want to ban ad blockers because of network effects among its users. They also assume that the platform can choose a different ad intensity for each type of user (e.g. show fewer ads to adblock users who whitelisted the site), which is an additional discriminatory tool platforms can use for their benefit. The difference with our paper is that we consider competition between platforms and we show that even when network effects are not present, and also platforms cannot directly discriminate users based on their type, ad blockers can still benefit them (due to a competition-softening effect). In another recent paper, Gritckevich et al. (2018) use an analytic model to study the business model of ad blockers and their relationship with publishers.

Several papers also study settings where a media provider has to decide between an adbased and a subscription-based strategy (Prasad et al., 2003; Peitz and Valletti, 2008; Tåg, 2009; Stühmeier and Wenzel, 2011; Vratonjic et al., 2013). Armstrong et al. (2009) study consumer protection policies and their impact on the consumers' incentives to become informed of market conditions. They show that when consumers are able to refuse marketing, price competition can decrease, which can harm consumers. Spam filters can also be considered a form of ad avoidance. Falkinger (2008) studies the equilibria in a model about spam filters with different levels of tolerance. In relation to ad annoyance, Goldstein et al. (2014) study the costs of annoying ads to publishers and users.

To the best of our knowledge, none of the previous work has considered ad avoidance from a perspective of a publisher who has the ability to prevent or limit it in a competitive setting. This is because prior work focused on traditional media providers, like TV stations, or direct marketing actions, like mail, calls, or email. Our setting, on the other hand, is inspired by web ads, ad blockers, and the available anti-adblock technology used by many websites today.³⁰ We extend the strategy space of publishers to include the most popular responses to ad blockers by websites, like adblock walls, pay walls, combinations of the two, allowing ad blockers, or paying for whitelisting services. This leads to quantitatively different results, with the general surprising conclusion that ad blockers can actually benefit both publishers and users in several different ways.

Outside the ad-avoidance literature, our results are also related (in terms of model mechanics) to price discrimination and price sensitivity (Corts, 1998; Desai, 2001; Desai and Purohit, 2004; Coughlan and Soberman, 2005; Pazgal et al., 2013). As an example of a related result, Jain (2008) uses a model of digital piracy to show that when more price-

³⁰https://www.wsj.com/articles/the-rise-of-the-anti-ad-blockers-1465805039 (accessed October 19, 2019)

sensitive consumers are the ones who copy software, then piracy can help firms.³¹ Shaffer and Zhang (1995) study the effects of price-discriminating customers by offering promotions based on their past purchase behavior and show that this can reduce the profits of the firms in a competitive environment.

3 Model

In this section, we describe the main model that we will use for the results in Section 4. In Sections 5.1 and 5.2, we extend this basic model by adding additional strategies to platforms' strategy space as well as additional segments of users. Table 1 contains a summary of the notation used throughout this paper.

3.1 Platform Model

There are two platforms, platform 1 and platform 2, competing over a set of users. Each platform can choose one of three different strategies, BAN, ALLOW, or FEE.

In the BAN strategy, the platform bans ad blockers by using an adblock wall. If a user with an ad blocker wants to access the site, she has to disable the ad blocker and see ads. The decision variable for a platform $i \in \{1, 2\}$ with the BAN strategy is the ad intensity $a_i \ge 0$. The revenue of a platform i with the BAN strategy is $r_i = (\text{mass of users who pick platform } i) \cdot a_i$.³²

In the ALLOW strategy, the platform allows the use of ad blockers. In this case, a user with an ad blocker can access the site without seeing any ads and the platform does not get any ad revenue from them. The decision variable for a platform $i \in \{1, 2\}$ with the ALLOW strategy is again the ad intensity $a_i \ge 0$. The revenue of a platform i with the ALLOW strategy is now $r_i = (\text{mass of users who pick platform } i$ and see ads $) \cdot a_i$.

In the FEE strategy, the platform offers content without any ads using a paywall. A user who wants to access the content has to pay a subscription fee for it. The decision variable for a platform $i \in \{1, 2\}$ with the FEE strategy is now the subscription fee $p_i \ge 0$. The revenue of a platform i with the FEE strategy is $r_i = (\text{mass of users who pick platform } i) \cdot p_i$.

3.2 User Model

We model users using a Hotelling line. We assume that the two platforms are positioned on the two endpoints of the interval [0, 1]. Each user draws a value x uniformly at random from [0, 1] that indicates her position in the interval. Users who are closer to a platform prefer that platform more than the other.

Any user's utility consists of three parts. The first part is some intrinsic value they have for accessing the platforms' service, e.g. for reading news, and it is independent of the

 $^{^{31}}$ In Section 8.1 we describe in more detail the similarities and the differences of the two models.

³²This simple form of revenue proportional to the ad intensity best models display advertising where the platform is compensated proportional to the number of ads shown under a cost-per-mille (thousand impressions) or CPM payment scheme. However, similar arguments as those in the paper can be used for more involved revenue schemes like CPC/cost-per-click and CPA/cost-per-action. The logic then is that removing more ad-sensitive users from the market can increase click-through-rates for platforms.

platform. We use the variable m to indicate this value. The second part is some intrinsic value each user has for the platform. This is where the Hotelling model is used. For a user at position x, this intrinsic value is 1 - x if they pick platform 1, and x if they pick platform 2. The third part is the disutility a user gets either from ads they have to see or from the price they have to pay when they choose a platform. We normalize the price sensitivity of users to 1 and we let their ad sensitivity vary. Throughout the paper we will see how the heterogeneity in the ad sensitivity between users can affect how platforms behave.

In the basic model, we assume that there are two segments of users. The first segment consists of users without an ad blocker.³³ This segment is of mass λ and its users have ad sensitivity β . The second segment consists of users with an ad blocker who use it whenever possible. This second segment is of mass μ and its users have ad sensitivity γ .³⁴ In other words, in this basic model, we assume some heterogeneity in the ad sensitivity between non-adblock and adblock users. Later, in extensions of the model, we will explore what happens when there is further heterogeneity in the ad sensitivity inside the segment of non-adblock users and inside the segment of adblock users.

Figure 2 summarizes the utility expressions for a user at position x who picked platform 1, based on the strategy the platform chose and the type of the user. To get the user utility for platform 2, we need to change (1 - x) to x, a_1 to a_2 , and p_1 to p_2 . Note the difference in ad sensitivity between the two types of users, and also that the adblock users do not suffer any disutility when the platform chooses the ALLOW strategy.³⁵

Platform 1		BAN	Allow	FEE
ADS	Non-adblock users (mass λ)	$m+(1-x)-\boldsymbol{\beta}\cdot\boldsymbol{a_1}$	$m+(1-x)-\boldsymbol{\beta}\cdot\boldsymbol{a}_1$	$m + (1-x) - \frac{p_1}{2}$
ADS	Adblock users (mass μ)	$m+(1-x)-\boldsymbol{\gamma}\cdot\boldsymbol{a_1}$	m+(1-x)	$m + (1-x) - \frac{p_1}{2}$

Figure 2: Utility of the user at position x who picks platform 1 based on the platform's strategy (columns) and the type of the user (rows).

Each user can pick at most one platform. We also assume that m, the intrinsic utility

 $^{^{33}}$ As mentioned earlier, some possible reasons that these users do not use an ad blocker are either ethical/moral, or because they do not know how to use one, or because they want to support the sites they visit.

³⁴Note that λ and μ in this model do not depend on the ad intensities a_1 and a_2 , i.e. the decision for users if they will install or not an ad blocker is exogenous. We can endogenize this decision by assuming that there is some cost for installing an ad blocker (e.g. learning cost) and letting users decide if they want to pay this cost based on their ad sensitivity and the ad intensity of the platform they visit. Even though the analysis becomes more complicated, we can show that the main results of the paper remain true (for different conditions). There are two reasons to avoid this direction. One is that to truly endogenize user's decision we need to consider other factors that may play a role in that decision, e.g. privacy concerns, but this is beyond the scope of this paper. The second and more important reason is that users usually do not decide to install an ad blocker based on the ad intensity of a single website. They are either installing an ad blocker and use it almost everywhere, no matter the ad intensity of each website, or they do not install one and see ads. Therefore, it is more realistic to assume that the decision is exogenous.

³⁵In Appendix A.1 we explore convex advertising cost functions of the form $\beta \cdot a_i^2$ and $\gamma \cdot a_i^2$. We show that the choice of the linear costs does not affect the main results of the paper.

for the service, is large enough so that every user picks at least one of the platforms.³⁶

3.3 Information Setting and Timeline

For simplicity we assume that all parameters are common knowledge. This is because any information uncertainty in the model would add extra complications without necessarily adding any insights regarding the effects of ad blockers. Moreover, in reality, even if a platform does not know the size of each segment of users or their ad sensitivity, there are ways to estimate these quantities. For example, they can use A/B testing with varying ad intensity to observe how users respond.

The timeline of the game is as follows. First, platforms choose the strategy they want to follow. This is the first step as this is the major decision platforms have to make. For example, using ads or subscription fees as their main business model usually means a different infrastructure for their website. Second, platforms decide the values of their decision variables (either ad intensity or price, depending on the strategy), as this is an easier decision to adjust. Third, users pick which platform to join based on the plan each platform offers so as to maximize their utility.

³⁶This assumption, which is standard in Hotelling models, reduces the number of cases we need to analyze. When it is not true, i.e. when there are users in the middle of the Hotelling interval who do not pick any platform, there is no interaction between platforms, making each platform act as a monopoly in their own part of the market. The fact that there is no competition then leads to less interesting results regarding ad blockers.

$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$
a_i Ad intensity of platform i (decision variable)
r_i Revenue of platform i
p_i Price of platform <i>i</i> (decision variable)
m Intrinsic value of users
λ Mass of non-adblock users
μ Mass of adblock users
β Ad sensitivity of non-adblock users
γ Ad sensitivity of adblock users
Section 5.1
ν Mass of non-adblock users with medium ad sensitivity
η Ad sensitivity of non-adblock users with medium ad sensitivity
Section 5.2
f Fee for the WHITELIST plan
ξ Mass of adblock users with medium ad sensitivity
ζ Ad sensitivity of adblock users with medium ad sensitivity
Section 6
q_i Quality of platform <i>i</i> 's content
c_i Coefficient of cost for generating content in platform i
<i>r</i> Coefficient of quality-based utility term for users
f_i Fraction of revenue that goes to content creators of platform i
π_i Profit of content creators of platform i
Section 7
j Type of a user: H or L for (H)igh-sensitive or (L)ow-sensitive
users
u_j Utility from advertising of user of type j
a^* Point where the advertising intensity maximizes users' utility
Δ Coefficient for the positive utility from advertising
<i>B</i> Coefficient for the negative utility from advertising of a user o
type L
Γ Coefficient for the negative utility from advertising of a user o
type H
t_i Threshold for advertising intensity after which a user of type
will start using an ad blocker

Table 1: Summary of notations in this paper.

3.4 Benchmark

In this section, we consider a world without ad blockers as a benchmark to compare the performance of the above model with ad blockers. This way, we can determine the effects of the presence of ad blockers on platforms and users.

When ad blockers do not exist, platforms can choose one of two different strategies: the

ADS plan or the FEE plan. A platform with the ADS plan offers its content for free to the users and its revenue comes from showing ads to them. In that case, since there are no ad blockers, every user has to be exposed to ads. A platform with the FEE plan offers its content without ads for a subscription fee.³⁷

		Platform 2	
		Ads	$\mathbf{F}\mathbf{E}\mathbf{E}$
Platform 1	Ads	$\frac{(\lambda+\mu)^2}{2(\beta\lambda+\gamma\mu)}, \frac{(\lambda+\mu)^2}{2(\beta\lambda+\gamma\mu)}$	$\frac{(\lambda+\mu)^2}{2(\beta\lambda+\gamma\mu)}, \frac{\lambda+\mu}{2}$
	Fee	$\frac{\lambda+\mu}{2}, \frac{(\lambda+\mu)^2}{2(\beta\lambda+\gamma\mu)}$	$\frac{\lambda+\mu}{2}, \frac{\lambda+\mu}{2}$

Table 2: Payoff matrix in the benchmark model.

If we analyze the game between the two platforms, we get the payoff matrix in Table 2.³⁸ In this game there are two symmetric equilibria, one where both platforms choose the ADS option and the other where both platforms choose the FEE option.³⁹ In Figure 3, we see the parameter regions of these two equilibria as a function of β (the ad sensitivity of the first segment of non adblock users) and $\frac{\gamma}{\beta}$ (the ratio of the ad sensitivities of the adblock users to that of the non-adblock users). As expected, when the ad sensitivities β and γ of users are low, platforms choose to show ads, while in the opposite case they decide to offer a subscription fee. The curve that separates the two equilibria is the line $\lambda + \mu = \beta \lambda + \gamma \mu$. The (ADS, ADS) strategy profile is an equilibrium iff $\lambda + \mu \geq \beta \lambda + \gamma \mu$, while the (FEE, FEE) strategy profile is an equilibrium iff $\lambda + \mu \leq \beta \lambda + \gamma \mu$.

4 Ad Blockers can be Beneficial

In this section we analyze the basic model and show how ad blockers can be beneficial for platforms in Subsection 4.1 and for users in Subsection 4.2.

4.1 Platforms' Welfare

Our first proposition shows that there is an equilibrium where both platforms allow ad blockers. In this equilibrium, even though both platforms get no revenue from adblock users and can ban ad blockers if they want, they still allow adblock users to access the content for free. Moreover, the revenue of the platforms when they allow ad blockers is sometimes higher than their revenue when they ban ad blockers or when they use a fee. As a result, the presence of ad blockers can make platforms better off compared to the benchmark model. (All proofs are in Appendix A.3.)

 $^{^{37}}$ The ADS plan is similar to the BAN plan of the main model and the FEE plan is the same as the one in the main model.

³⁸For the proof, see the first part of the proof of Proposition 1 in Appendix A.3.

³⁹Asymmetric equilibria can occur only in the degenerate case where $\lambda + \mu = \beta \lambda + \gamma \mu$. In that case, all possible pair of strategies give the same revenue to the platforms. Since this is a region of measure zero in the parameter space, we ignore it for the remainder of the discussion.



Figure 3: Equilibria regions of the benchmark model for $\lambda = 1$ and $\mu = 2$.

Proposition 1. There is an equilibrium where both platforms ALLOW ad blockers. In this equilibrium, when β is sufficiently low and $\frac{\gamma}{\beta}$ is sufficiently high, platforms are better off than they would be if ad blockers did not exist.



Figure 4: Illustration of the ad sensitivities of the two segments of users in the main model.

The intuition for this result is as follows. Let's assume that the ad sensitivity γ of the adblock users is larger than β , the ad sensitivity of non-adblock users (Figure 4). When platforms ban ad blockers, they show ads to both segments of users. However, the competition between the two platforms for the ad-sensitive segment will drive the optimal ad intensity of both platforms down. As a result, platforms can end up with low ad-revenue. On the other hand, when platforms allow ad blockers, they do not get any revenue from the segment of adblock users, but they have to compete only for the segment of non-adblock users that are less ad-sensitive. This allows them to increase the advertising intensity, which can result in higher ad-revenue.

The higher the difference in the ad sensitivities of the two segments, the more incentive platforms have to filter ad-sensitive users from the market and focus only on the users with low ad sensitivity. As a result, higher $\frac{\gamma}{\beta}$ makes the ALLOW option more attractive to platforms than the BAN option. Moreover, the lower the ad sensitivity of those who see ads in the ALLOW strategy (i.e., low β), the more attractive the ALLOW plan becomes over the FEE plan. This gives the two conditions in Proposition 1.

If we analyze all possible pair of strategies for the two platforms, we get the payoff matrix in Table 3. As in the benchmark model, there are three symmetric equilibria in this game, one where both platforms ban ad blockers, one where they allow ad blockers, and one where they choose a fee. A difference is that now there are regions in the parameter space with more than one equilibria. In Figure 5, we can see the equilibria regions as a function of β (the ad sensitivity of non-adblock users) and $\frac{\gamma}{\beta}$ (the ratio of the ad sensitivity of adblock users over the ad sensitivity of non-adblock users). In regions with more than one equilibria, we list all of them and the first one in the list is the best one for platforms.

	BAN	Allow	$\mathbf{F}\mathbf{E}\mathbf{E}$
Ban	$rac{(\lambda+\mu)^2}{2(eta\lambda+\gamma\mu)}, rac{(\lambda+\mu)^2}{2(eta\lambda+\gamma\mu)}$	$rac{(3\lambda+2\mu)^2(eta\lambda+\gamma\mu)}{2(3eta\lambda+4\gamma\mu)^2}, rac{\lambda(3eta\lambda+eta\mu+2\gamma\mu)^2}{2eta(3eta\lambda+4\gamma\mu)^2}$	$rac{(\lambda+\mu)^2}{2(eta\lambda+\gamma\mu)},rac{\lambda+\mu}{2}$
Allow	$\frac{\lambda(3\beta\lambda+\beta\mu+2\gamma\mu)^2}{2\beta(3\beta\lambda+4\gamma\mu)^2},\frac{(3\lambda+2\mu)^2(\beta\lambda+\gamma\mu)}{2(3\beta\lambda+4\gamma\mu)^2}$	$rac{\lambda}{2eta},rac{\lambda}{2eta}$	$\frac{9\lambda(\lambda+\mu)^2}{2\beta(3\lambda+4\mu)^2}, \frac{(\lambda+\mu)(3\lambda+2\mu)^2}{2(3\lambda+4\mu)^2}$
Fee	$rac{\lambda+\mu}{2}, rac{(\lambda+\mu)^2}{2(eta\lambda+\gamma\mu)}$	$rac{(\lambda+\mu)(3\lambda+2\mu)^2}{2(3\lambda+4\mu)^2}, rac{9\lambda(\lambda+\mu)^2}{2eta(3\lambda+4\mu)^2}$	$rac{\lambda+\mu}{2},rac{\lambda+\mu}{2}$

Table 3: Payoff matrix in the main model.

If we examine the plot in Figure 5 from the bottom towards the top, we see that when $\frac{\gamma}{\beta}$ is low, i.e. when adblock users are less ad-sensitive than non-adblock users or when the ad sensitivity of the two segments is similar, both platforms prefer to ban ad blockers. However, as $\frac{\gamma}{\beta}$ increases, the ALLOW option becomes more and more attractive for platforms. Thus, first we get a region where both BAN and ALLOW are equilibria but BAN is the better one for platforms, then a region where both are equilibria but ALLOW is better, and finally a region where ALLOW is the unique equilibrium.

If we now review the plot from the right towards the left, we see that as β (the ad sensitivity of non-adblock users, who see ads when platforms allow ad blockers) becomes lower, the ALLOW option becomes more and more attractive to platforms than the FEE option. This is because the lower the β , the more the platforms can increase the ad intensity, and as a result their ad revenue.

4.2 User Welfare

The next proposition shows that total user welfare goes up when platforms allow ad blockers. In other words, not only do platforms benefit from the presence of ad blockers, but users could benefit too. Moreover, there are cases where total user welfare goes up and no user is worse off compared to the benchmark.

Proposition 2. When both platforms allow ad blockers, total user welfare is higher than in the other two equilibria. Moreover, there are regions in the parameter space where platforms are better off, total user utility goes up, and no user is worse off compared to a world without ad blockers.



Figure 5: Equilibria regions of the main model for $\lambda = 1$ and $\mu = 2$. When there are two equilibrium strategies in the same region, the first one (in bold) is better for platforms.

The total user utility in the BAN equilibrium is the same as the one in the FEE equilibrium, while in the ALLOW equilibrium it is higher. The reason is that when both platforms allow ad blockers the segment of adblock users get no disutility from ads since they block them. This improves the overall user utility even if non-adblock users are sometimes worse off because they have to see more ads.

The regions mentioned in the second part of Proposition 2 are the regions labeled "ALLOW" and "ALLOW, FEE" in Figure 5. In these same two regions in the benchmark model, there is only one equilibrium where both platforms use a subscription fee. The disutility non-adblock users get from the fee in the benchmark model is the same as the disutility they get from ads when platforms allow ad blockers. As a result, their utility in these regions is the same as it was in the benchmark, while every other user and the platforms are better off.

5 Additional Plans

In this section, we investigate the effect of adding two different options to the strategy space of the platforms: an ADS OR FEE option that lets users choose between watching ads or paying for an ad-free plan, and a WHITELIST option to paying a fee to the ad blocker to whitelist their ads among users employing the ad blocker. Even when the ADS OR FEE option is available to platforms, we demonstrate that our main counterintuitive finding that the ALLOW option continues to be an equilibrium still holds in certain parameter regions. When WHITELIST is allowed, we show that this is an equilibrium option for both platforms, and perhaps even more surprisingly, despite the payment to the ad blocker, this improves their revenues compared to a world with no ad blockers at all. To show these results, we refine the set of adblock users or non-adblock users into two further subsegments with differing ad sensitivities.

5.1 The ADS OR FEE Plan

The reason the ALLOW plan can benefit platforms in the basic model is that it provides a natural way for them to discriminate users with different ad sensitivities. Ideally platforms would like to be able to choose a different ad intensity for segments of users with different ad sensitivity. When they cannot do that, ad blockers provide an exogenous mechanism to achieve a similar effect. Ad-sensitive users self select themselves out of the market by using ad blockers, and this way they help not only themselves but also the competing platforms.

However, there is another natural way to "discriminate" users without the help of ad blockers that a lot of web sites currently use.⁴⁰ This is by letting users choose between two different options: either get free access to the site with ads (no ad blockers are allowed) or pay a fee for an ad-free version of the site (behind a paywall). We call this new plan the ADS OR FEE plan.

The ADS OR FEE plan is a combination of the BAN and the FEE plan from the basic model that tries to achieve the best of both worlds. The rationale is that users who are not very ad-sensitive will decide to see ads, while ad-sensitive users will choose the fee option. This solves the problem the BAN strategy had in the basic model of the ad-sensitive segment forcing platforms to decrease ad intensity. With the ADS OR FEE strategy platforms can choose a high ad intensity for non-adblock users and also make sensitive adblock users pay a fee to access the content. Thus, this new strategy has the benefits of the ALLOW strategy plus some potential extra revenue from adblock users that platforms could not get earlier by allowing ad blockers.

The main question we want to answer in this section is whether the ADS OR FEE strategy always dominates the benefits of allowing ad blockers. In other words, does the addition of the ADS OR FEE plan wipe out the beneficial effects of the ALLOW strategy and prevents it from ever becoming an equilibrium. To answer this question, we extend the model from Section 3 by adding the ADS OR FEE strategy to platforms' strategy space. If a platform chooses this strategy they will have two decision variables, an ad intensity a_i and a price p_i . Users who pick a platform with the ADS OR FEE plan will choose between being exposed to ads or paying the price, based on which option gives them higher utility.

We also consider a more refined view of the user ad sensitivities by adding a third segment of users to the model.⁴¹ This segment has mass ν and is made of non-adblock users with ad sensitivity η with $\beta \leq \eta \leq \gamma$ (Figure 6). The reason we add this third segment of users is that the comparison of the ADS OR FEE plan with the ALLOW plan will depend on how heterogeneous the ad sensitivity of non-adblock users is, which we refine by splitting into two subsegments. Then, by allowing $\frac{\eta}{\beta}$ to change, we can compare the two plans.

The following proposition shows that even after we add the ADS OR FEE strategy to the game, ad blockers can still be beneficial for platforms. There is still an equilibrium where

⁴⁰Some notable examples are Wired, Bild, and Business Insider.

⁴¹As we will see later, the addition of the extra segment is necessary for the results in this section.



Figure 6: Illustration of the ad sensitivities of the three segments of users in the model with the additional ADS OR FEE strategy.

platforms allow ad blockers, while sometimes this is the unique equilibrium and sometimes it is the best among others, including one where both platforms choose the ADS OR FEE plan.

Proposition 3. When we add the ADS OR FEE plan to platforms' strategy space, there is still an equilibrium where both platforms ALLOW ad blockers. In this equilibrium, platforms are sometimes better off than in a world without ad blockers. There are regions in the parameter space where this is the unique equilibrium and regions where it is the best equilibrium for platforms among others.

The reason the ADS OR FEE strategy does not always dominate the ALLOW strategy and there are still cases platforms get higher revenue by allowing ad blockers is the following. Let's assume that γ , the ad sensitivity of the third segment, is very large compared to β and η , the ad sensitivities of the first and the second segment (see Figure 6 for an illustration). The platform then prefers to avoid showing adds to the third segment of users to avoid having to lower its ad intensity. There are two ways to achieve that, either with the ALLOW strategy (where the third segment will use ad blockers) or with the ADS OR FEE strategy (where the third segment will choose the fee option due to their high ad sensitivity). Let's consider the scenario where the platform uses the ADS OR FEE strategy and η is sufficiently high so that the second segment of users prefers the fee over the ads. That is good for the platform because high η means that they want to avoid showing add to the second segment as well. But now let's assume that η starts decreasing towards β . This means that the advertising revenue the platform could extract from the second segment if they were forced to see ads goes up. Therefore, from the perspective of the platform, at some point this advertising revenue will exceed the revenue it gets from the fee by the second segment. That does not necessarily mean though that the second segment would also prefer to see ads instead of paying the fee. It can be the case that even at that point, the second segment prefers the fee option. Mathematically, for an advertising intensity a and a fee p, it can be a > p (the revenue the platform extracts from the second segment in the two cases), while $\eta \cdot a < p$ (the disutility of the second segment in the two cases). This can happen when η is sufficiently small.

When this happens, we have a situation where the platform wants the second segment of

users to see ads, but in the ADS OR FEE strategy those users choose the fee option. Then, the ALLOW strategy provides a solution for the platform. Since there is no fee option in ALLOW, the second segment will see ads. Even though with ALLOW the platform loses the fee revenue from the third segment, as long as β and η are sufficiently small, the extra profit compensates for those losses.

We now describe the conditions under which ALLOW is the best strategy for platforms. For the ALLOW strategy to be better than FEE, we want the users who see ads in ALLOW to have relatively low ad sensitivity, i.e. we want low β and η . To make ALLOW better than BAN, we want the ad sensitivity of those who see ads in BAN but not in ALLOW to be higher than those who see ads in both, i.e. we want high $\frac{\gamma}{\beta}$ and $\frac{\gamma}{\eta}$. Finally, for ALLOW to be better than ADS OR FEE, we want those who see ads in ALLOW but do not see ads in ADS OR FEE to have similar ad sensitivity as those who see ads in both, i.e. we want low $\frac{\eta}{\beta}$. This is because otherwise ADS OR FEE would be the better option to separate non-adblock users of different ad sensitivities. In other words, for ALLOW to be the best strategy, we want the subsegments of non adblock users to be nearly homogeneous in their sensitivities and well separated from the sensitivity of the adblock users.

After we analyze all possible strategy combinations for the two platforms, we obtain the payoff matrix in Table 4. As before, there are four different symmetric equilibria, one for each strategy that is available to the platforms. In Figures 7, 8, and 9, we can see the equilibria regions for different parameters. To make the pictures a bit simpler, when there are more than one equilibria we list the best one for platforms. Thus, the regions where ALLOW is the unique equilibrium are subregions of the regions labeled ALLOW in the plots and near the borders there are multiple equilibria, one for each region that shares the border.



Figure 7: Equilibria regions of the model with the ADS OR FEE strategy for $\lambda = \mu =$ $\nu = 1$ and $\frac{\eta}{\beta} = \frac{3}{2}$. When there are more than one equilibria, only the best one for platforms is listed.



Figure 8: Equilibria regions of the model with the ADS OR FEE strategy for $\lambda = \mu =$ $\nu = 1$ and $\frac{\gamma}{\beta} = 4$. When there are more than one equilibria, only the best one for platforms is listed.

In Figure 7, we can see that ALLOW is the preferred option for platforms when β is low and $\frac{\gamma}{\beta}$ is high. Moreover, ADS OR FEE is better than ALLOW for relatively higher values of η and because $\frac{\eta}{\beta}$ is fixed in that plot, that means higher values of β make ADS OR FEE better than ALLOW.

Similarly, we can see in Figure 8 that ALLOW is the preferred option when β is low and $\frac{\gamma}{\eta}$ is high. Moreover, when we compare ALLOW with ADS OR FEE, higher η is better for ADS OR FEE, which means lower $\frac{\gamma}{\eta}$ is better for it.



Figure 9: Equilibria regions of the model with the ADS OR FEE strategy for $\lambda = \mu = \nu = 1$ and $\frac{\gamma}{\eta} = 4$. When there are more than one equilibria, only the best one for platforms is listed.

Finally, in Figure 9, we see that lower $\frac{\eta}{\beta}$ makes ALLOW better than ADS OR FEE. This plot is also an example where the BAN region disappears. Even though BAN is an equilibrium when β and $\frac{\eta}{\beta}$ are low, it is never the best for the particular choice of parameter values (γ is too high compared to the other ad sensitivities).

5.2 Acceptable Ads and Whitelisting

In 2011, Eyeo, the company that developed Adblock Plus, the most popular adblock extension for browsers, started a program called the "Acceptable Ads initiative".⁴² They set a list of criteria of what are considered acceptable ads based on placement, size, etc., and ads that complied with those criteria would be whitelisted by default in their ad blocker. Large companies, like Google, Microsoft, and Amazon were paying monthly fees to Eyeo to partic-

⁴²https://adblockplus.org/acceptable-ads (accessed October 19, 2019)

ipate in this program and let their ads pass through the ad blocker.^{43,44} These whitelisting services were also the main source of revenue for Adblock Plus.⁴⁵ In 2016, Eyeo extended this program by launching their own ad marketplace where they also started selling "acceptable" ads to publishers.^{46,47}

The controversial nature of these moves by Adblock Plus created a lot of backlash. As an example, the CEO of the Interactive Advertising Bureau (IAB) characterized Adblock Plus as an "extortion-based business" and their actions "unethical and immoral".^{48,49} The main argument is that first the company created an ad blocker that allowed millions of users to block ads on websites and now the same company charges money from advertisers and publishers to unblock their ads. This could be seen as a form of blackmail by those publishers and advertisers who now have to share part of their revenue with the ad-blocker company.

The key question we address in this context is whether this new option where platforms have to pay the ad-blocker company to whitelist their ads could ever be beneficial for them. In the presence of ad blockers, we may expect that sometimes platforms will have to follow this strategy, first if ad blockers hurt their ad revenue a lot, and second to get an advantage over competitors with blocked ads. However, more importantly, when they follow this strategy, how does their revenue change when compared to a world without ad blockers? In other words, can the option of paying the ad blocker to let their ads go through ever benefit them over the benchmark model without ad blockers where their ads were shown "for free"?

To answer this question we use another extension of the basic model. First, we add a new strategy to platforms' strategy space, called WHITELIST. When a platform chooses this WHITELIST option, they allow ad blockers and at the same time they pay a fee $f \ge 0^{50}$ to the ad-blocker company to whitelist their ads by default. As in the real world, some users who do not like this whitelist feature and do not want to watch even "acceptable" ads, have the option to disable the feature and remove all ads. We again consider three segments of users, but this time refining the segment of adblock users; our model supposes one segment of non-adblock users of mass λ with ad sensitivity β and two segments of adblock users. The first segment of adblock users are those who are fine with the whitelisting program and keep the default whitelist of "acceptable" ads. That segment is of mass ξ with ad sensitivity ζ . The second segment of adblock users are those who are against all ads, remove the default whitelist, and as a result block all ads. That segment is of mass μ with ad sensitivity γ . We do not assume any relationship between β , ζ , and γ , but as we show next, the most

⁴³https://www.ft.com/content/80a8ce54-a61d-11e4-9bd3-00144feab7de (accessed October 19, 2019)

⁴⁴http://www.businessinsider.com/google-microsoft-amazon-taboola-pay-adblock-plus-tostop-blocking-their-ads-2015-2 (accessed October 19, 2019)

⁴⁵https://adblockplus.org/about#monetization (accessed October 19, 2019)

⁴⁶https://www.theverge.com/2016/9/13/12890050/adblock-plus-now-sells-ads (accessed October 19, 2019)

⁴⁷https://acceptableads.com/en/solutions/ (accessed October 19, 2019)

⁴⁸https://techcrunch.com/2016/05/09/interactive-adverting-bureau-ceo-adblock-plus-isan-extortion-based-business/ (accessed October 19, 2019)

⁴⁹http://www.businessinsider.com/interactive-advertising-bureau-comments-on-ad-blockplus-2016-1 (accessed October 19, 2019)

⁵⁰In the case of Adblock Plus and their program, this fee is zero for small entities without a lot of ad impressions and strictly positive for larger entities.



Figure 10: Illustration of the ad sensitivities of the three segments of users in the model with the WHITELIST strategy.

interesting results occur when $\beta \leq \gamma$ and $\zeta \leq \gamma$.

The next proposition answers the question above by showing that there is an equilibrium where both platforms choose the WHITELIST plan and that this equilibrium is sometimes better for platforms compared to the benchmark model with no ad blockers.

Proposition 4. When we add the WHITELIST plan to platforms' strategy space, there is an equilibrium where both platforms choose the WHITELIST option. In this equilibrium, platforms are sometimes better off than they would be if ad blockers did not exist.

To understand the intuition behind the proposition, consider the case where adblock users are more ad-sensitive than non-adblock users and that those adblock users who remove all ads are even more ad-sensitive than those who keep the ad blocker's whitelist, i.e. $\beta \leq \zeta \leq \gamma$ (Figure 10). The main idea is that the WHITELIST option can help platforms separate the two types of adblock users.

A platform with the BAN plan chooses some advertising intensity a > 0 and shows ads to all three segments of users (Figure 10). With the ALLOW plan, they choose some advertising intensity a' > a and show ads only to the first segment of users. As we have seen in the basic model, sometimes ALLOW is better because of the high a' and sometimes BAN is better because the platforms get ad-revenue from more users. With the WHITELIST plan, the platform chooses some advertising intensity a'' with a < a'' < a' and they show ads to the first two segments of users, the non-adblockers and the adblocker users who keep the whitelist. What happens is that this middle ground between BAN and ALLOW sometimes provides more revenue than the other two, i.e. showing ads to exactly two segments with medium ad intensity is better than showing ads to all three segments with low ad intensity or to just one segment with high ad intensity.

The conditions for which WHITELIST is the best option for platforms are the following. First, we want a sufficiently small fee f, otherwise WHITELIST will become a bad option because it is expensive. Second, we want the ad sensitivity of those who see ads in the WHITELIST plan to be low to make WHITELIST better than FEE, i.e. we want low β and ζ . Third, we want those who see ads in BAN but not in WHITELIST to have comparatively higher ad sensitivity, i.e. we want high $\frac{\gamma}{\beta}$ and $\frac{\gamma}{\zeta}$. Finally, to make WHITELIST better than ALLOW, we want the ad sensitivity of those who see ads in WHITELIST and not in ALLOW to have similar ad sensitivity to those who see ads in both, otherwise ALLOW that separates them would be better. Therefore, we also want low $\frac{\zeta}{\beta}$. In other words, for WHITELIST to be the best strategy, we want the subsegments of adblock users to be heterogeneous and well separated in their sensitivities, while the sensitivity of the lower segment among these is comparable to that of non-adblock users.

After we analyze all possible pair of strategies for platforms, we get the payoff matrix in Table 5. In this game, there are four symmetric equilibria, one for each strategy. In Figures 11, 12, and 13, we see the equilibria regions for different parameter values. As before, to make plots a bit simpler, when there are more than one equilibria we list only the best one for platforms. The regions where each equilibrium is unique are subregions of those labeled in the plots.



Figure 11: Equilibria regions of the model with the WHITELIST strategy for f = 0, $\lambda = \mu = \xi = 1$, and $\zeta = \frac{1}{2}$. When there are more than one equilibria, only the best one for platforms is listed.



Figure 12: Equilibria regions of the model with the WHITELIST strategy for f = 0, $\lambda = \mu = \xi = 1$, and $\frac{\gamma}{\beta} = \frac{7}{2}$. When there are more than one equilibria, only the best one for platforms is listed.

In Figure 11, we see that the WHITELIST plan is preferred by platforms when $\frac{\gamma}{\beta}$ is high and β is low. When we compare the WHITELIST plan with the ALLOW plan, WHITELIST is better for low $\frac{\zeta}{\beta}$. In this particular plot ζ is fixed, so low $\frac{\zeta}{\beta}$ means high β . Therefore, there is a lower and an upper bound for β to make WHITELIST the best option.

In Figure 12, we see that the WHITELIST plan is preferred when $\frac{\gamma}{\zeta}$ is high and β is low. To understand why the BAN and the ALLOW regions are in the order they are, let's assume that β is fixed. Since in that plot $\frac{\gamma}{\beta}$ is also fixed, that means γ is fixed. BAN is better than ALLOW when $\frac{\zeta}{\beta}$ is low, which means when ζ is low, which means when $\frac{\gamma}{\zeta}$ is high.

In Figure 13, we see that the WHITELIST plan is preferred for medium values of $\frac{\zeta}{\beta}$ and low β . We know that WHITELIST is better than ALLOW for lower values of $\frac{\zeta}{\beta}$. For the



Figure 13: Equilibria regions of the model with the WHITELIST strategy for f = 0, $\lambda = \mu = \xi = 1$, and $\frac{\gamma}{\zeta} = 2$. When there are more than one equilibria, only the best one for platforms is listed.

comparison between WHITELIST and BAN, let's again assume that β is fixed. Higher $\frac{\zeta}{\beta}$ means higher ζ and since $\frac{\gamma}{\zeta}$ for that plot is fixed, that means higher γ . But γ is the ad sensitivity of those who see ads in the BAN plan and do not see ads in the WHITELIST plan. Therefore, higher γ makes WHITELIST the better option.

In April of 2017, there were surprising reports that Google is planning to create their own built-in ad blocker for the Chrome browser.⁵¹ This ad blocker will remove only "unaccept-able" ads from web pages. In other words, Google wants to implement a similar program like the "Acceptable Ads initiative" by Adblock Plus. This is further evidence that Google realizes the benefits of an ad blocker with a whitelisting feature, even though Google itself depends heavily on advertising. Thus, instead of letting third parties implement such a feature and take part of their ad revenue, Google might prefer to do it on its own and thus exercise more control of the ad-blocker market.

6 Quality of Content and Content Creators

An important feature of many internet platforms that affects users decision of which platform to join is the quality of content. Some platforms generate their own content, while others depend on third parties to generate content for them.

YouTube is an example of a platform that does not generate its own content. Instead

⁵¹https://www.wsj.com/articles/google-plans-ad-blocking-feature-in-popular-chromebrowser-1492643233 and https://www.theverge.com/2017/4/19/15365572/google-ad-blockingfeature-chrome-browser (accessed October 19, 2019)

it depends on content creators to create and upload videos on the website that other users watch. For a very long time YouTube was dependent solely on advertising as its main source of revenue. However, recently it started offering a subscription plan to its users (named YouTube Red) for an ad-free version of YouTube with some additional exclusive content. Any revenue YouTube gets from advertising and from subscriptions is shared with the content creators. YouTube gets 45% of the revenue, while content creators get the remaining 55%.⁵² We examine the question of how the quality of content is affected by the advent of ad blockers under a revenue-sharing model, like the one YouTube implemented. To do that, we extend the basic model by adding content creators.

We assume that each platform has its own content creators. Content creators in platform $i \in \{1, 2\}$ have as decision variable the quality of content q_i and they incur some cost $c_i \cdot q_i^2$ to generate content of this quality.⁵³ User utility is the same as in the basic model with an additional quality-based utility term of $r \cdot q_i$ for platform i, i.e. higher quality of content in a platform means higher utility for users in that platform. Each platform has also a fixed fraction f_i that determines how they split the revenue with the content creators. The profit for content creators in platform i is $\pi_i = f_i \cdot (\text{total revenue}) - c_i \cdot q_i^2$, while the profit for platform i is $r_i = (1 - f_i) \cdot (\text{total revenue})$. Finally, the timeline of the game is the same as before with the addition that in the second step content creators also decide the value of q_i to maximize their profit, before users choose between the platforms.

We can show that all the results of the basic model are robust under this extension with content creators. More specifically, there is an equilibrium where platforms allow ad blockers and all platforms, content creators, and users are better off compared to a world without ad blockers. The fact that content creators can be better off when ad blockers are allowed causes an increase in the quality of content. This is the main result in the next proposition.

Proposition 5. When ad blockers are allowed by platforms, the quality of content is higher for sufficiently high $\frac{\gamma}{\beta}$ and sufficiently low β . This is an additional benefit for users when ad blockers are allowed, whose welfare can increase even more than the increase with ad blockers in the basic model without content creators.

Indeed, for the case of symmetric platforms, total user welfare is the same in this extension and in the basic model when platforms BAN ad blockers or when they use a FEE. However, when platforms ALLOW ad blockers, user welfare is higher in this extension than in the basic model. This is because when platforms BAN ad blockers or use a FEE, all the extra value that is generated by the quality of content goes to the platforms and the content creators in the form of increased ad intensity or price. However, when platforms ALLOW ad blockers, the higher quality of content allows this extra value to be shared with users who benefit even more with ad blockers.

⁵²https://support.google.com/youtube/answer/6204741 (accessed October 19, 2019)

⁵³The convex cost function is a natural choice for increasing quality which has diminishing returns to the effort by the creator.

7 Concave Utility from Advertising and Endogenous Decision by Users

In Section 3, we made two simplifying assumptions in our model. The first one is that users without an ad blocker always suffer a negative linear utility from advertising. The second one is that the ad sensitivity of users is perfectly correlated with ad-blocker usage, namely users with an ad blocker are only those with high ad sensitivity. In this section, we show that neither of these assumptions are necessary for our main results.

Here we assume that the utility consumers receive from advertising follows a concave function. The idea is that when advertising is served in low quantities it can actually be useful for consumers for getting information about products that might interest them. On the other hand, when consumers are exposed to a lot of ads, advertising starts to become annoying and results in a negative utility for consumers. Therefore, to remove the first simplifying assumption we'll assume that users have a utility from advertising like the one in Figure 14.



Figure 14: Form of the utility users receive from advertising as a function of the advertising intensity.

To remove the second simplifying assumption, we will now assume that all users have access to an ad blocker and they will use it whenever they want. Users might also have varying ad sensitivity, expressed by their utility function. There are low-sensitive users with utility from advertising given by

$$u_L(a_i) = \begin{cases} \Delta a_i, & \text{if } a_i \leq a^*, \\ \Delta a^* + B(a^* - a_i), & \text{otherwise.} \end{cases}$$

and high-sensitive users with utility from advertising given by

$$u_H(a_i) = \begin{cases} \Delta a_i, & \text{if } a_i \le a^*, \\ \Delta a^* + \Gamma(a^* - a_i), & \text{otherwise}. \end{cases}$$

where $\Gamma \geq B \geq 0$ and $\Delta, a^* \geq 0$. Note that when $a_i = 0$ users receive zero utility from advertising, when $a_i > (1 + \frac{\Delta}{B}) a^* =: t_L$ low-sensitive users receive negative utility from advertising (which means that they will use an ad blocker), and when $a_i > (1 + \frac{\Delta}{\Gamma}) a^* =: t_H$ high-sensitive users receive negative utility from advertising (which means that they will use an ad blocker). Note also that $t_H \leq t_L$.

The rest of the model is similar to our main model. Each platform will choose an advertising intensity a_i and decide if it will ALLOW ad blockers or BAN them.⁵⁴ Then users will decide which platform to join and if they will use an ad blocker or not (when it is allowed). The following proposition is an analog of Proposition 1 for this model.

Proposition 6. When users have a concave utility from advertising and their decision to use an ad blocker or not is endogenous, there is an equilibrium where both platforms ALLOW ad blockers. In this equilibrium, when B is sufficiently low and $\frac{\Gamma}{B}$ is sufficiently high, platforms are better off than they would be if ad blockers did not exist.

The main idea of this proposition is that sometimes when ad blockers are allowed, platforms choose, in equilibrium, advertising intensities a_1 and a_2 in the region (t_H, t_L) , where high-sensitive users use ad blockers while low-sensitive users do not. If platforms ban ad blockers, high-sensitive users will stop using them but at the same time the competition between platforms for those users will increase and the ad intensities will decrease. This leads to making ALLOW a better option for platforms than BAN.

8 Conclusion

8.1 Contribution to the Literature

The mechanism underlying the result of Proposition 1 is similar to the main result of Jain (2008) in the context of digital piracy. In his paper, Jain shows that firms' profits can be higher when they do not enforce copyright protection as opposed to when they do. The reason is that by allowing price-sensitive consumers to copy software, they are able to increase the price for the rest of the consumers. Here we want to point out four important differences of our paper compared with Jain (2008), as well as some extra insights into this principle that the unique features of online advertising can provide.

The first difference comes from the model itself. If we restrict the strategy space of the platforms to only two strategies, BAN and ALLOW, then we can see the resemblance of the two results by thinking of the BAN strategy as the analog of enforcing copyright protection and the ALLOW strategy as the analog of no copyright protection. The addition of the FEE strategy in our model however, which does not have an analog in digital piracy under this comparison, has some extra implications. The main one is the result of Proposition 2, where we show that allowing ad blockers can not only improve platforms' revenues but it can also be a Pareto improvement over the no-adblock world, i.e. user welfare goes up while no user is worse off. This result is only possible because of the addition of the FEE strategy in our

 $^{^{54}}$ For simplicity we ignore the FEE strategy here, but all the ideas stay the same if we add this strategy as well.

models, as the Pareto improvement only occurs in a region where the equilibrium in the no-adblock world is for both platforms to use the FEE strategy. In other words, the fact that users can not only get a disutility from advertising but also from price provides some extra insights due to the interaction between the two, that is not present in Jain (2008).

A second difference can be seen in Section 5.1. Besides the fact that an analog of the ADS OR FEE strategy does not exist in the piracy world, an analog of Proposition 3 is not obvious even if one existed. More specifically, Jain's setting cannot explain why the ALLOW strategy can sometimes be better than both the BAN and the ADS OR FEE strategy at the same time, without the added heterogeneity in the ad sensitivity of the users. While ALLOW is better than BAN for the same reasons as in Proposition 1 and ADS OR FEE is better than ALLOW for similar reasons, it is not clear if ALLOW can ever be better than both BAN and ADS OR FEE at the same time. In fact, a hypothetical analog of the ADS OR FEE strategy in Jain's model would always dominate the ALLOW strategy under the conditions of Proposition 1. Therefore, the heterogeneity in the ad-sensitivity of users that we add in the model by including an extra segment of users is necessary to show that ad blockers are beneficial even after the inclusion of the ADS OR FEE strategy.

Moreover, the ADS OR FEE strategy is a very relevant strategy in online advertising today, as more and more websites implement a version of it as a way to deal with ad blockers. Therefore, studying it in addition to the result of the main model is an important contribution to the literature. In our opinion, it is very surprising that allowing ad blockers can still be beneficial after the inclusion of the ADS OR FEE strategy.

The WHITELIST strategy of Section 5.2 is also very unique to online advertising, which points out the third difference. As WHITELIST is another very relevant strategy nowadays, it is interesting to see how it interacts with the rest of the strategies and to show that even if platforms are asked to pay a fee to ad-blocker companies, ad blockers can still benefit them.

A fourth important difference is illustrated in Section 7. In the piracy model, no matter if someone has high or low price sensitivity, paying a lower price is always preferable to paying a higher price. This means that in Jain's model, if we give the piracy option to every user, then everyone will pirate and as a result firms would never want to allow piracy. That's why a necessary assumption in that model is that only high price-sensitive users have the ability to copy software. But to justify this assumption, some exogenous reasoning is required. In Proposition 6 we show that in the advertising world things are different. Due to the nature of advertising, even if every user has the ability to block ads, it can still be beneficial for platforms to allow ad blockers.

In addition to these differences, in the next section we provide some managerial implications of our results that as far as we know are new in the literature.

8.2 Managerial Implications

Our analysis leads to several managerial implications. It can provide websites with some general guidelines regarding the plan they should choose based on how heterogeneous their visitors are in their ad sensitivity. In Figure 15, we exhibit a decision diagram summarizing our findings.

The decision flowchart contains four questions in increasing order of refinement. If the users are generally very ad-sensitive, then the platform cannot expect to receive a lot of ad



Figure 15: Flowchart for platforms to decide which strategy to follow based on the heterogeneity in the ad sensitivity of their user base.

revenue from them, so it is better to choose a subscription based plan with a fee (FEE). Otherwise, the platform can benefit from serving add to the users. If the ad sensitivities of non-adblock and adblock users are similar (or if non-adblock users are more ad-sensitive), then ad blockers cannot help the platform and it is better to ban them with an adblock wall (BAN). If, on the other hand, adblock users are more ad-sensitive than non-adblock users, then the platform would be better off with a plan that filters out the very ad-sensitive adblock users. The third question is about the ad sensitivity of non-adblock users. If it is very heterogeneous, then a plan that offers options to the users like the ADS OR FEE plan can help the platform filter out the very ad-sensitive non-adblock users. If non-adblock users have homogeneous ad sensitivity and adblock users are more ad-sensitive than non-adblock users, then allowing ad blockers can be beneficial. The heterogeneity of adblock users plays a role here. If adblock users are homogeneous then just allowing ad blockers can be enough (ALLOW), but if they are very heterogeneous then a whitelist option on top of allowing ad blockers can be the better plan (WHITELIST), since it filters out only the very ad-sensitive part of adblock users and keeps the rest, even if that means the platform has to pay a fee to the ad blocker for whitelisting its ads.

The way these findings can be used is the following. First, a platform can run a few tests with varying types of adblock walls or messages to the users (as many web platforms already do) in order to estimate the ad sensitivities of the various user segments. Then it can use the insights from our analysis to decide the ideal plans to offer.

8.3 Summary

Ad blockers initiated an existential crisis in the world of online platforms subsisting on advertising. While most speculations point to a grim outlook for advertisers and platforms as a result of ad blockers, our results offer an alternative view that might offer a glimmer of hope for the whole ecosystem, by arguing that ad blockers could actually be beneficial overall. We suggest several ways in which ad blocking can be beneficial. First, they can make the market more efficient by filtering out ad-sensitive users for more intense or targeted ad serving on the rest. Second, ad-sensitive users can benefit because they can remove ads that annoy them from web sites. Third, ad blockers can also help regulate the ad industry through a whitelisting program of acceptable ads. Finally, a more efficient market can also result in an increase in content quality of web sites, which is an additional benefit for users.

A few years ago, when ad blockers started rising, publishers and advertisers were terrified of their implications for the future of the online ad industry. However, today we see that many of them choose a more friendly approach towards ad blockers. News like the recent plan of Google to create its own ad blocker in its Chrome browser,⁵⁵ shows that the industry has started realizing the potential benefits of ad blocking and decided to make it an ally instead of an enemy. As in many existential crises, the result could be rewarding.

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⁵⁵https://www.engadget.com/2017/06/01/google-chrome-ad-blocking-baked-in-2018/ (accessed October 19, 2019)

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A Appendix

A.1 Convex Advertising Cost Functions

In this section, we explore what happens when we change the advertising cost functions of the main model from linear to convex. The intuition behind the convex advertising cost is that as the advertising intensity becomes higher and higher, the annoyance of the users increases at a higher rate.

More specifically, we change the advertising cost functions from $\beta \cdot a$ and $\gamma \cdot a$ to $\beta \cdot a^2$ and $\gamma \cdot a^2$, respectively, while keeping everything else in the model the same. Due to the higherdegree equations arising in the analysis, deriving analytical solutions is no longer tractable. To test the robustness of our main results, we tried several numerical examples to generate plots similar to the one in Figure 5. As expected, the results are consistent in the updated model.

In Figures 16, 17, and 18, we can see that for high $\frac{\gamma}{\beta}$ and low β , it is an equilibrium for both platforms to allow ad blockers. Moreover, there are regions where (ALLOW, ALLOW) is the unique equilibrium. Finally, we can see that as the mass of adblock users, μ , increases compared to the mass of non-adblock users, λ , the region where (ALLOW, ALLOW) is an equilbrium becomes smaller.

A.2 More Benefits of Ad Blockers

A.2.1 Non-adblock Users Can Benefit As Well

In Proposition 2, we showed that ad blockers, in addition to benefiting platforms, can also benefit adblock users without making non-adblock users worse off. In this section, we present an argument that non-adblock users can indirectly benefit from ad blockers as well. As a result, there are situations where everyone is strictly better off with ad blockers than without.

The main idea is the following. Suppose that there is a firm with a new product in the market. The firm needs to advertise the product to consumers and make them aware of it. Therefore, the firm wants to discourage people from using ad blockers. In order to do so, it can offer a price discount announced through the ads. Those without ad blockers will



Figure 16: Equilibria regions of the model with the convex advertising costs, for $\lambda = 1$ and $\mu = 0.5$.

see ads, learn about the discount and benefit from it, while those with ad blockers will be unaware of it. Knowing this, people have an incentive to stop using ad blockers.

The equilibrium outcome will be that while some consumers with high ad sensitivity will keep using ad blockers, others with lower ad sensitivity will decide to see ads and benefit from the discounted price. In contrast, in a world without ad blockers, the firm would not offer a discounted price because everyone would see the ad anyway. In this way, non-adblock users indirectly benefit from the existence of ad blockers because they can take advantage of discounts that would not exist without ad blockers.

Of course, we still need to show that all the above materializes in an equilibrium, i.e., it is optimal for the firm to offer a lower price when ad blockers exist and a higher price when ad blockers do not exist. To do this, we consider the following simple model.⁵⁶

The decision variable for the firm is the price of the product p. The consumers are of varying ad sensitivities s. The consumers do not know anything about the new product, but they have some expectation about their valuation v for the product and the price p. Based on s, and their expectations for v and p, each consumer decides if they will use an ad blocker or not.

If someone does not use an ad blocker, then they see an ad for the product, their valuation v is realized, the price p is revealed to them, and they decide if they will buy the product or not. Consumers who decided to use an ad blocker will not learn about the product and as a result they will not buy it.⁵⁷

 $^{^{56}}$ We could in theory extend the main model of the paper (Section 3) here by including on top of everything else a firm that provides the ads to the platforms, but the model will become unnecessarily complicated. For illustrative purposes, we consider a simpler model here.

⁵⁷In Section A.2.2 we remove this assumption and adblock users are able to buy as well.



Figure 17: Equilibria regions of the model with the convex advertising costs, for $\lambda = \mu = 1$.



Figure 18: Equilibria regions of the model with the convex advertising costs, for $\lambda = 1$ and $\mu = 2$.

As a benchmark model we use the same framework but without ad blockers, i.e. everyone will see the ad independently of their sensitivity s.

For simplicity, let's also assume that both the ad sensitivity s and the valuation v for each consumer are drawn from a uniform distribution in [0, 1]. Moreover, the disutility a consumer with ad sensitivity s will get from seeing an ad is $\sigma \cdot s$, for some coefficient $\sigma \geq 0$. The following proposition summarizes the results for this model

The following proposition summarizes the results for this model.

Proposition 7. In comparison to the benchmark, the following can be observed in the equilibria of the model with ad blockers.

- 1. The price is lower. In other words, the firm offers a discount to prevent people from using ad blockers.
- 2. Consumers who do not use an ad blocker are better off. This is a result of the lower price offered by the firm to them.
- 3. Consumers who use an ad blocker are better off. This is because those consumers have high ad sensitivity and they don't have to incur the cost of ads.

Notice that in the model above, the firm is worse off with ad blockers, because it will sell the product to fewer people at a lower price. However, we can enrich the model to show that the firm can sometimes benefit too. We consider one such extension in Section A.2.2.

In summary, in this section we provide one more potential benefit of ad blockers for users. Besides the obvious benefit for adblock users, non-adblock users benefit too because they can take advantage of discounts that would not exist without ad blockers.

A.2.2 The Firm Can Benefit As Well

In Section A.2.1 the firm was worse off after the introduction of ad blockers, but this does not have to be the case. In this section, we show one way in which the firm can benefit too.

One of the assumptions in Section A.2.1 was that adblock users don't buy the product because they don't see an ad and as a result they don't learn about it. In reality though there is another way to learn about the product without seeing ads. This is by actively searching for more information about the product in order to learn the actual valuation for it. If consumers decide to learn this way, then they have to incur some search cost (or learning cost).

In this section we consider such a possibility. More specifically, consumers can do one of three things:

- They can choose to not use an ad blocker and be exposed to ads. In this case they will learn about the product from an ad.
- They can choose to use an ad blocker and pay a search cost ψ to learn about the product by themselves.
- They can choose to use an ad blocker and not learn about the product. In this case they don't incur any cost and their valuation is 0.

When ψ is constant, the model and the results are very similar to Section A.2.1. For this reason, here we will allow some heterogeneity in the search cost, i.e. ψ can be different for different consumers.

Now we can see why the firm can sometimes be better off with ad blockers even though it has to lower its price. There are some consumers with high ad sensitivity and low search cost. In a world without ad blockers, the firm would ideally like to separate such consumers from the rest (and let them search and learn about the product by themselves), and show ads to the remaining. But there is no mechanism to do this. Ad blockers provide such a mechanism, since these consumers will use ad blockers and the firm does not have to incur the cost of showing ads to them. Therefore, when there is a sufficiently high number of consumers with high ad sensitivity and low search cost, and the cost of advertising for the firm is also sufficiently high, the firm can be better off as well in the world with ad blockers.

To formalize this argument, let's assume that ψ and s are related through some function $\psi = \psi(s)$. For simplicity of exposition, we consider the linear function $\psi(s) = \phi_1 + \phi_2 \cdot s$, for some constants ϕ_1 , ϕ_2 .⁵⁸ We also assume that there is some cost for the firm to advertise to consumers. To advertise to a mass z of consumers the firm has a cost of $w \cdot z$ for some constant $w \ge 0$.

Proposition 8. There is an equilibrium where in addition to consumers, the firm is strictly better off with ad blockers.

The equilibrium of Proposition 8 occurs when ϕ_2 is sufficiently negative (which means that consumers with high ad sensitivity have lower search cost and vice versa) and w is sufficiently high (so as to make the firm want to stop advertising to some consumers) but not too high (which would prevent the firm from advertising at all).

 $^{^{58}\}text{The only}$ assumption for ϕ_1 and ϕ_2 is that they should keep $\psi(s)$ positive.

A.3 Analyses and Proofs

Proof of Proposition 1. We consider first the scenario where ad blockers do not exist (benchmark model). This is when platforms have only two available strategies: the ADS strategy where everyone who access the websites has to see ads, and the FEE strategy where users have to pay a fee to access the site.

In this scenario, we start by considering four possible cases, one for each combination of plans chosen by the two platforms. For each one of these cases, we will find how the users react and then which of those cases end up in an equilibrium.

1. Both platforms use ADS. The indifferent user among those without ad blockers is the one at position x_N that is the solution to the equation $m + 1 - x_N - \beta a_1 = m + x_N - \beta a_2$, i.e. $x_N = \frac{1+\beta(a_2-a_1)}{2}$.

The indifferent user among those with ad blockers (when they are available) is the one at position x_A that is the solution to the equation $m + 1 - x_A - \gamma a_1 = m + x_A - \gamma a_2$, i.e. $x_A = \frac{1 + \gamma (a_2 - a_1)}{2}$.

Therefore, the expected market share of platform 1 is $z_1 = \lambda x_N + \mu x_A$, while the expected market share of platform 2 is $z_2 = \lambda(1 - x_N) + \mu(1 - x_A)$. The profit for platform 1 is then z_1a_1 and the profit for platform 2 is z_2a_2 . Thus, to find the advertising intensities a_1 , a_2 in the equilibrium, we need to solve the system $\frac{\partial(z_1a_1)}{\partial a_1} = \frac{\partial(z_2a_2)}{\partial a_2} = 0$. The solution is $a_1 = a_2 = \frac{\lambda + \mu}{\beta \lambda + \gamma \mu}$. From this, we get that the profit for both platforms is equal to $\frac{(\lambda + \mu)^2}{2(\beta \lambda + \gamma \mu)}$. 2. Both platforms have a subscription FEE. In this case, both types of users have similar

2. Both platforms have a subscription FEE. In this case, both types of users have similar payoff function, thus the indifferent user for both types is the one at position x that is the solution to the equation $m + 1 - x - p_1 = m + x - p_2$, i.e. $x = \frac{1 + p_2 - p_1}{2}$.

Therefore, the expected market share of platform 1 is $z_1 = (\lambda + \mu)x$, while the expected market share of platform 2 is $z_2 = (\lambda + \mu)(1 - x)$.

The profit for platform 1 is then z_1p_1 and the profit for platform 2 is z_2p_2 . Thus, to find the prices p_1 , p_2 in the equilibrium, we need to solve the system $\frac{\partial(z_1p_1)}{\partial p_1} = \frac{\partial(z_2p_2)}{\partial p_2} = 0$. The solution is $p_1 = p_2 = 1$. From this, we get that the profit for both platforms is equal to $\frac{\lambda + \mu}{2}$.

3. First platform uses ADS, while the second uses a FEE. The indifferent user among nonadblock users is the one at position x_N that is the solution to the equation $m+1-x_N-\beta a_1 = m+x_N-p_2$, i.e. $x_N = \frac{1+p_2-\beta a_1}{2}$.

The indifferent user among adblock users is the one at position x_A that is the solution to the equation $m + 1 - x_A - \gamma a_1 = m + x_A - p_2$, i.e. $x_A = \frac{1 + p_2 - \gamma a_1}{2}$.

The expected market share of platform 1 is $z_1 = \lambda x_N + \mu x_A$, while the expected market share of platform 2 is $z_2 = \lambda(1 - x_N) + \mu(1 - x_A)$. The profit for platform 1 is then z_1a_1 and the profit for platform 2 is z_2p_2 . Thus, to find the advertising intensity a_1 and the price p_2 in the equilibrium, we need to solve the system $\frac{\partial(z_1a_1)}{\partial a_1} = \frac{\partial(z_2p_2)}{\partial p_2} = 0$. The solution is $a_1 = \frac{\lambda + \mu}{\beta \lambda + \gamma \mu}$ and $p_2 = 1$.⁵⁹ From this, we get that the profit for platform 1 is $\frac{(\lambda + \mu)^2}{2(\beta \lambda + \gamma \mu)}$, while the profit for platform 2 is $\frac{\lambda + \mu}{2}$.

⁵⁹Note that the indifferent user for those who do not use ad blockers is the one at position $x_N = \frac{\beta\lambda - \beta\mu + 2\gamma\mu}{2(\beta\lambda + \gamma\mu)}$, which is always at most 1, but we also need this to be non-negative. Therefore, we need that $\beta\lambda + 2\gamma\mu \ge \beta\mu$. Similarly, for the indifferent user for those with ad blockers, we need the inequality $2\beta\lambda + \gamma\mu \ge \gamma\lambda$. Due to symmetry, we get the same conditions from the fourth case of the proof as well.

4. First platform uses a FEE, while the second uses ADS. This case is similar to the previous case. The profit for platform 1 is $\frac{\lambda+\mu}{2}$, while the profit for platform 2 is $\frac{(\lambda+\mu)^2}{2(\beta\lambda+\gamma\mu)}$. The following payoff matrix summarizes the four cases above.

		Platform 2	
		Ads	$\mathbf{F}\mathbf{E}\mathbf{E}$
Platform 1	Ads Fee	$\frac{\frac{(\lambda+\mu)^2}{2(\beta\lambda+\gamma\mu)},\frac{(\lambda+\mu)^2}{2(\beta\lambda+\gamma\mu)}}{\frac{\lambda+\mu}{2},\frac{(\lambda+\mu)^2}{2(\beta\lambda+\gamma\mu)}}$	$\frac{\frac{(\lambda+\mu)^2}{2(\beta\lambda+\gamma\mu)},\frac{\lambda+\mu}{2}}{\frac{\lambda+\mu}{2},\frac{\lambda+\mu}{2}}$

We observe that

- (ADS, ADS) is an equilibrium iff $\lambda + \mu \ge \beta \lambda + \gamma \mu$;
- (FEE, FEE) is an equilibrium iff $\lambda + \mu \leq \beta \lambda + \gamma \mu$;
- (ADS, FEE) and (FEE, ADS) are equilibria iff $\lambda + \mu = \beta \lambda + \gamma \mu$.

Now we consider the scenario of the main model, where ad blockers are introduced and the second type of users can use them if they are allowed. The BAN strategy of the main model is similar to the ADS strategy of the benchmark, since every user has to see ads in both of them. Therefore, for the analysis of the main model we can use the four cases we considered in the benchmark and add to them five more cases for the strategy profiles that include the ALLOW strategy.

5. Both platforms ALLOW ad blockers. The indifferent user among those who do not use ad blockers is the one at position x_N that is the solution to the equation $m + 1 - x_N - \beta a_1 = m + x_N - \beta a_2$, i.e. $x_N = \frac{1 + \beta(a_2 - a_1)}{2}$.

The indifferent user among those who use ad blockers is the one at position x_A that is the solution to the equation $m + 1 - x_A = m + x_A$, since now this type of users can use ad blockers to avoid ads. It is $x_A = \frac{1}{2}$.

The expected market share of platform 1 is $z_1 = \lambda x_N$, while the expected market share of platform 2 is $z_2 = \lambda(1 - x_N)$. The profit for platform 1 is then z_1a_1 and the profit for platform 2 is z_2a_2 . Thus, to find the advertising intensities a_1, a_2 in the equilibrium, we need to solve the system $\frac{\partial(z_1a_1)}{\partial a_1} = \frac{\partial(z_2a_2)}{\partial a_2} = 0$. The solution is $a_1 = a_2 = \frac{1}{\beta}$. From this, we get that the profit for both platforms is equal to $\frac{\lambda}{2\beta}$.

6. First platform ALLOWS ad blockers, while the second uses BAN. The indifferent user among those who do not use ad blockers is the one at position x_N that is the solution to the equation $m + 1 - x_N - \beta a_1 = m + x_N - \beta a_2$, i.e. $x_N = \frac{1 + \beta (a_2 - a_1)}{2}$.

The indifferent user among those with ad blockers is the one at position x_A that is the solution to the equation $m + 1 - x_A = m + x_A - \gamma a_2$. It is $x_A = \frac{1 + \gamma a_2}{2}$.

The expected market share of platform 1 is $z_1 = \lambda x_N$, while the expected market share of platform 2 is $z_2 = \lambda(1-x_N) + \mu(1-x_A)$. The profit for platform 1 is then z_1a_1 and the profit for platform 2 is z_2a_2 . Thus, to find the advertising intensities a_1 , a_2 in the equilibrium, we need to solve the system $\frac{\partial(z_1a_1)}{\partial a_1} = \frac{\partial(z_2a_2)}{\partial a_2} = 0$. The solution is $a_1 = \frac{3\beta\lambda + \beta\mu + 2\gamma\mu}{\beta(3\beta\lambda + 4\gamma\mu)}$ and

 $a_2 = \frac{3\lambda + 2\mu}{3\beta\lambda + 4\gamma\mu}$.⁶⁰ From this, we get that the profit for platform 1 is equal to $\frac{\lambda(3\beta\lambda + \beta\mu + 2\gamma\mu)^2}{2\beta(3\beta\lambda + 4\gamma\mu)^2}$, while the profit for platform 2 is $\frac{(3\lambda + 2\mu)^2(\beta\lambda + \gamma\mu)}{2(3\beta\lambda + 4\gamma\mu)^2}$.

7. First platform ALLOWs ad blockers, while the second uses FEE. The indifferent user among non-adblockers is the one at position x_N that is the solution to the equation $m + 1 - x_N - \beta a_1 = m + x_N - p_2$, i.e. $x_N = \frac{1 + p_2 - \beta a_1}{2}$.

The indifferent user among those who use ad blockers is the one at position x_A that is the solution to the equation $m + 1 - x_A = m + x_A - p_2$. It is $x_A = \frac{1+p_2}{2}$.

The expected market share of platform 1 is $z_1 = \lambda x_N$, while the expected market share of platform 2 is $z_2 = \lambda(1 - x_N) + \mu(1 - x_A)$. The profit for platform 1 is z_1a_1 and the profit for platform 2 is z_2p_2 . Thus, to find the advertising intensity a_1 and the price p_2 in the equilibrium, we need to solve the system $\frac{\partial(z_1a_1)}{\partial a_1} = \frac{\partial(z_2p_2)}{\partial p_2} = 0$. The solution is $a_1 = \frac{3(\lambda+\mu)}{\beta(3\lambda+4\mu)}$ and $p_2 = \frac{3\lambda+2\mu}{3\lambda+4\mu}$.⁶¹ From this, we get that the profit for platform 1 is equal to $\frac{9\lambda(\lambda+\mu)^2}{2\beta(3\lambda+4\mu)^2}$, while the profit for platform 2 is $\frac{(\lambda+\mu)(3\lambda+2\mu)^2}{2(3\lambda+4\mu)^2}$.

8. First platform BANs ad blockers, while the second ALLOWs them. This is similar to case 6. The profit for platform 1 is equal to $\frac{(3\lambda+2\mu)^2(\beta\lambda+\gamma\mu)}{2(3\beta\lambda+4\gamma\mu)^2}$, while the profit for platform 2 is $\frac{\lambda(3\beta\lambda+\beta\mu+2\gamma\mu)^2}{2(3\beta\lambda+4\gamma\mu)^2}$.

 $\frac{(\lambda + \mu)^2}{2\beta(3\beta\lambda + 4\gamma\mu)^2}$ 9. First platform uses a FEE, while the second ALLOWs ad blockers. This is similar to case 7. The profit for platform 1 is equal to $\frac{(\lambda + \mu)(3\lambda + 2\mu)^2}{2(3\lambda + 4\mu)^2}$, while the profit for platform 2 is $\frac{9\lambda(\lambda + \mu)^2}{2\beta(3\lambda + 4\mu)^2}$.

Summarizing all of the above, we get the following payoff matrix for the two platforms.

	BAN	Allow	$\mathbf{F}\mathbf{E}\mathbf{E}$
Ban	$rac{(\lambda+\mu)^2}{2(eta\lambda+\gamma\mu)}, rac{(\lambda+\mu)^2}{2(eta\lambda+\gamma\mu)}$	$\frac{(3\lambda+2\mu)^2(\beta\lambda+\gamma\mu)}{2(3\beta\lambda+4\gamma\mu)^2},\frac{\lambda(3\beta\lambda+\beta\mu+2\gamma\mu)^2}{2\beta(3\beta\lambda+4\gamma\mu)^2}$	$rac{(\lambda + \mu)^2}{2(eta\lambda + \gamma\mu)}, rac{\lambda + \mu}{2}$
Allow	$\frac{\lambda(3\beta\lambda+\beta\mu+2\gamma\mu)^2}{2\beta(3\beta\lambda+4\gamma\mu)^2},\frac{(3\lambda+2\mu)^2(\beta\lambda+\gamma\mu)}{2(3\beta\lambda+4\gamma\mu)^2}$	$rac{\lambda}{2eta},rac{\lambda}{2eta}$	$\frac{9\lambda(\lambda+\mu)^2}{2\beta(3\lambda+4\mu)^2},\frac{(\lambda+\mu)(3\lambda+2\mu)^2}{2(3\lambda+4\mu)^2}$
$\mathbf{F}\mathbf{E}\mathbf{E}$	$rac{\lambda+\mu}{2}, rac{(\lambda+\mu)^2}{2(eta\lambda+\gamma\mu)}$	$rac{(\lambda+\mu)(3\lambda+2\mu)^2}{2(3\lambda+4\mu)^2}, rac{9\lambda(\lambda+\mu)^2}{2eta(3\lambda+4\mu)^2}$	$rac{\lambda+\mu}{2},rac{\lambda+\mu}{2}$

We observe that (ALLOW, ALLOW) is an equilibrium iff the following two conditions hold.

$$\frac{\lambda}{2\beta} \ge \frac{(\lambda+\mu)(3\lambda+2\mu)^2}{2(3\lambda+4\mu)^2} \quad \text{and} \quad \frac{\lambda}{2\beta} \ge \frac{(3\lambda+2\mu)^2(\beta\lambda+\gamma\mu)}{2(3\beta\lambda+4\gamma\mu)^2}$$

These two conditions are equivalent to

$$\beta \leq \frac{\lambda(3\lambda + 2\mu)^2}{(\lambda + \mu)(3\lambda + 2\mu)^2} \quad \text{and} \quad \frac{\lambda + \mu \cdot \frac{\gamma}{\beta}}{\left(3\lambda + 4\mu \cdot \frac{\gamma}{\beta}\right)^2} \leq \frac{\lambda}{(3\lambda + 2\mu)^2}$$

The function $g(x) = \frac{\lambda + \mu \cdot x}{(3\lambda + 4\mu \cdot x)^2}$ is decreasing, therefore (ALLOW, ALLOW) is an equilibrium for low enough β and high enough $\frac{\gamma}{\beta}$.

⁶⁰Note that the indifferent user for those who do not use ad blockers is the one at position $x_N = \frac{3\beta\lambda+\beta\mu+2\gamma\mu}{6\beta\lambda+8\gamma\mu}$, which is always non-negative, but we also need this to be at most 1. Therefore, we need that $3\beta\lambda+6\gamma\mu\geq\beta\mu$. Similarly, for the indifferent user for those who use ad blockers, we need the inequality $3\beta\lambda+2\gamma\mu\geq3\gamma\lambda$. We get the same conditions for the symmetric case 8.

⁶¹The indifferent user for those who do not use ad blockers is the one at position $x_N = \frac{3(\lambda+\mu)}{\beta(6\lambda+8\mu)}$, which is always non-negative and at most 1. Therefore, we do not need any extra inequality here. Similarly, for the indifferent user for those who use ad blockers, we do not need any extra condition either. The same is true for the symmetric case 9.

Moreover, if

=

$$\frac{\lambda}{2\beta} \geq \frac{(\lambda+\mu)^2}{2(\beta\lambda+\gamma\mu)} \qquad \text{and} \qquad \frac{\lambda}{2\beta} \geq \frac{\lambda+\mu}{2}$$

then the profits of the two platforms in the (ALLOW, ALLOW) equilibrium are larger than their profits in the (BAN, BAN) and (FEE, FEE) equilibria. These conditions are equivalent to $\frac{\gamma}{\beta} \geq 2 + \frac{\mu}{\lambda}$ and $\beta \leq \frac{\lambda}{\lambda + \mu}$, so again when β is low enough and $\frac{\gamma}{\beta}$ is high enough. \Box

Proof of Proposition 2. When both platforms choose ALLOW, the user utility is

$$\begin{split} \lambda \left(\int_0^{\frac{1}{2}} \left(m + 1 - x - \beta \cdot \frac{1}{\beta} \right) dx + \int_{\frac{1}{2}}^1 \left(m + x - \beta \cdot \frac{1}{\beta} \right) dx \right) \\ + \mu \left(\int_0^{\frac{1}{2}} \left(m + 1 - x \right) dx + \int_{\frac{1}{2}}^1 \left(m + x \right) dx \right) \\ \lambda \left(m - \frac{1}{4} \right) + \mu \left(m + \frac{3}{4} \right). \end{split}$$

When both platforms choose BAN, the user utility is

$$\begin{split} \lambda \left(\int_{0}^{\frac{1}{2}} \left(m+1-x-\beta \cdot \frac{\lambda+\mu}{\beta\lambda+\gamma\mu} \right) dx + \int_{\frac{1}{2}}^{1} \left(m+x-\beta \cdot \frac{\lambda+\mu}{\beta\lambda+\gamma\mu} \right) dx \right) \\ &+ \mu \left(\int_{0}^{\frac{1}{2}} \left(m+1-x-\gamma \cdot \frac{\lambda+\mu}{\beta\lambda+\gamma\mu} \right) dx + \int_{\frac{1}{2}}^{1} \left(m+x-\gamma \cdot \frac{\lambda+\mu}{\beta\lambda+\gamma\mu} \right) dx \right) \\ &= \lambda \left(m+\frac{3}{4} - \frac{\beta(\lambda+\mu)}{\beta\lambda+\gamma\mu} \right) + \mu \left(m+\frac{3}{4} - \frac{\gamma(\lambda+\mu)}{\beta\lambda+\gamma\mu} \right) \\ &= (\lambda+\mu) \left(m - \frac{1}{4} \right). \end{split}$$

When both platforms choose FEE, the user utility is

$$\begin{split} \lambda \left(\int_{0}^{\frac{1}{2}} \left(m + 1 - x - 1 \right) dx + \int_{\frac{1}{2}}^{1} \left(m + x - 1 \right) dx \right) \\ &+ \mu \left(\int_{0}^{\frac{1}{2}} \left(m + 1 - x - 1 \right) dx + \int_{\frac{1}{2}}^{1} \left(m + x - 1 \right) dx \right) \\ &= \lambda \left(m - \frac{1}{4} \right) + \mu \left(m - \frac{1}{4} \right) \\ &= (\lambda + \mu) \left(m - \frac{1}{4} \right). \end{split}$$

Note that the total user utility is the same when both platforms choose BAN or both platforms choose FEE, and it is higher when both platforms choose ALLOW.

Moreover, the utility of users without ad blockers is the same when both platforms choose ALLOW or both platforms choose FEE, while the utility of users with ad blockers is always higher when both platforms choose ALLOW. This implies that in the regions where platforms were using FEE in the benchmark but ALLOW in the main model, no user is worse off with ALLOW. \Box

Proof of Proposition 3. This proof requires to consider 16 cases, one for each strategy profile, similar to the cases of the proof of Proposition 1 but now with three segments of users. Since the proof is very repetitive, we illustrate here only one of the cases and then we provide the payoff matrix we obtain after the full analysis.

Both platforms use the ADS OR FEE plan. This case has several subcases based on what option (between ads or fee) each segment of users picks. Since $\beta \leq \eta \leq \gamma$, there are four subcases for platform 1:

- $p_1 < \beta a_1 \le \eta a_1 \le \gamma a_1$
- $\beta a_1 \le p_1 \le \eta a_1 \le \gamma a_1$
- $\beta a_1 \leq \eta a_1 < p_1 \leq \gamma a_1$
- $\beta a_1 \leq \eta a_1 \leq \gamma a_1 < p_1$

In the first subcase, every user in platform 1 prefers to pay the fee, therefore this subcase is as if platform 1 had chosen the FEE plan, and we can ignore it here.⁶² Similarly, the fourth subcase is as if platform 1 had chosen the BAN plan, and we can ignore it too. This leaves us with two subcases for platform 1 and two similar subcases for platform 2. Therefore, there are four possible strategy profiles we need to analyze. For brevity, we show only the two symmetric ones here.

1. $\beta a_1 \leq p_1 \leq \eta a_1 \leq \gamma a_1$ and $\beta a_2 \leq p_2 \leq \eta a_2 \leq \gamma a_2$. The indifferent user among the first segment of non-adblockers is the one at position x_N that is the solution to the equation $m + 1 - x_N - \beta a_1 = m + x_N - \beta a_2$, i.e. $x_N = \frac{1 + \beta a_2 - \beta a_1}{2}$.

The indifferent user among the second segment of non-adblockers is the one at position $x_{N,2}$ that is the solution to the equation $m + 1 - x_{N,2} - p_1 = m + x_{N,2} - p_2$, since those non-adblock users pay the fee. It is $x_{N,2} = \frac{1+p_2-p_1}{2}$.

The indifferent user among adblock users is the one at position x_A that is the solution to the equation $m + 1 - x_A - p_1 = m + x_A - p_2$. It is $x_A = \frac{1 + p_2 - p_1}{2}$.

The expected mass of users in platform 1 who see ads is $z_{1,a} = \lambda x_N$, while the expected mass of users in platform 1 who pay the fee is $z_{1,p} = \nu x_{N,2} + \mu x_A$. Similarly, the expected mass of users in platform 2 who see ads is $z_{2,a} = \lambda(1-x_N)$, while the expected mass of users in platform 2 who pay the fee is $z_{2,p} = \nu(1-x_{N,2}) + \mu(1-x_A)$.

The profit for platform 1 is $z_{1,a}a_1 + z_{1,p}p_1$ and the profit for platform 2 is $z_{2,a}a_2 + z_{2,p}p_2$. Thus, to find the advertising intensities a_1 , a_2 and the prices p_1 , p_2 in the equilibrium, we need to solve the system $\frac{\partial(z_{1,a}a_1+z_{1,p}p_1)}{\partial a_1} = \frac{\partial(z_{1,a}a_1+z_{1,p}p_1)}{\partial p_1} = \frac{\partial(z_{2,a}a_2+z_{2,p}p_2)}{\partial a_2} = \frac{\partial(z_{2,a}a_2+z_{2,p}p_2)}{\partial p_2} = 0$. The solution is $a_1 = a_2 = \frac{1}{\beta}$ and $p_1 = p_2 = 1$. From this, we get that the profit for both platforms is equal to $\frac{\lambda + \beta(\mu + \nu)}{2\beta}$.

Note that the solution we found satisfy the inequalities of this subcase. Therefore, this subcase gives us a possible equilibrium.

⁶²Note that the ADS OR FEE plan is more general than the FEE and the BAN plan, as the platform can always make ad intensity or price equal to $+\infty$, forcing all of its users to either pay the fee or see ads respectively. To avoid confusion throughout the paper, when we say that a platform uses the ADS OR FEE plan, we actually mean that some of its users see ads and some of its users decide to pay the fee. When we say that a platform uses the FEE plan, we actually mean that it either uses the ADS OR FEE plan with ad intensity equal to $+\infty$ or it uses the actual FEE plan with no option for ads. Similarly for the BAN strategy.

2. $\beta a_1 \leq \eta a_1 < p_1 \leq \gamma a_1$ and $\beta a_2 \leq \eta a_2 < p_2 \leq \gamma a_2$. The indifferent user among the first segment of non-adblock users is the one at position x_N that is the solution to the equation $m + 1 - x_N - \beta a_1 = m + x_N - \beta a_2$, i.e. $x_N = \frac{1 + \beta a_2 - \beta a_1}{2}$.

The indifferent user among the second segment of non-adblock users is now the one at position $x_{N,2}$ that is the solution to the equation $m + 1 - x_{N,2} - \eta a_1 = m + x_{N,2} - \eta a_2$, since in these subcase those non-adblock users see ads. It is $x_{N,2} = \frac{1 + \eta a_2 - \eta a_1}{2}$.

The indifferent user among adblock users is the one at position x_A that is the solution to the equation $m + 1 - x_A - p_1 = m + x_A - p_2$, i.e. $x_A = \frac{1 + p_2 - p_1}{2}$.

The expected mass of users in platform 1 who see ads is $z_{1,a} = \lambda x_N + \nu x_{N,2}$, while the expected mass of users in platform 1 who pay the fee is $z_{1,p} = \mu x_A$. Similarly, the expected mass of users in platform 2 who see ads is $z_{2,a} = \lambda(1-x_N) + \nu(1-x_{N,2})$, while the expected mass of users in platform 2 who pay the fee is $z_{2,p} = \mu(1-x_A)$.

The profit for platform 1 is $z_{1,a}a_1 + z_{1,p}p_1$ and the profit for platform 2 is $z_{2,a}a_2 + z_{2,p}p_2$. Thus, to find the advertising intensities a_1 , a_2 and the prices p_1 , p_2 in the equilibrium, we need to solve the system $\frac{\partial(z_{1,a}a_1+z_{1,p}p_1)}{\partial a_1} = \frac{\partial(z_{1,a}a_1+z_{1,p}p_1)}{\partial p_1} = \frac{\partial(z_{2,a}a_2+z_{2,p}p_2)}{\partial a_2} = \frac{\partial(z_{2,a}a_2+z_{2,p}p_2)}{\partial p_2} = 0$. The solution is $a_1 = a_2 = \frac{\lambda+\nu}{\beta\lambda+\eta\nu}$ and $p_1 = p_2 = 1$. From this, we get that the profit for both platforms is equal to $\frac{\lambda^2+\beta\lambda\mu+2\lambda\nu+\eta\mu\nu+\nu^2}{2(\beta\lambda+\eta\nu)}$. Note however, that the solution we found this time does not satisfy the inequalities of

Note however, that the solution we found this time does not satisfy the inequalities of this subcase, because $\eta a_1 = \frac{\eta \lambda + \eta \nu}{\beta \lambda + \eta \nu} \ge \frac{\beta \lambda + \eta \nu}{\beta \lambda + \eta \nu} = 1 = p_1$. Therefore, this subcase does not give an equilibrium.

After we analyze all possible cases and subcases, we obtain the payoff matrix in Table 4. From that matrix, we can also obtain the conditions under which both platforms choose ALLOW as their equilibrium strategy.

An example where (ALLOW, ALLOW) is the unique equilibrium is for $\lambda = \mu = \nu = 1$, $\beta = \frac{1}{4}$, $\eta = \frac{3}{10}$, and $\gamma = 2$. In that case, the payoff matrix becomes

	Ban	Allow	${ m Fee}$	Ads or Fee
Ban	1.8, 1.8	0.88, 1.4	1.8, 1.5	1.8, 2.2
Allow	1.4, 0.88	3.6, 3.6	2.9, 0.96	3.1, 2.2
$\mathbf{F}\mathbf{E}\mathbf{E}$	1.5, 1.8	0.96, 2.9	1.5, 1.5	1.5, 3
Ads or Fee	2.2, 1.8	2.2, 3.1	3, 1.5	3, 3

Proof of Proposition 4. As in the proof of Proposition 3, this proof requires to consider all the 16 possible strategy combinations for the platforms. For brevity we show only one in detail here and then we provide the payoff matrix with the result of all cases.

Both platforms use the WHITELIST plan. The indifferent user among the non-adblock users is the one at position x_N that is the solution to the equation $m + 1 - x_N - \beta a_1 = m + x_N - \beta a_2$, i.e. $x_N = \frac{1 + \beta a_2 - \beta a_1}{2}$.

The indifferent user among the first segment of adblock users, who keep the whitelist, is the one at position $x_{A,2}$ that is the solution to the equation $m+1-x_{A,2}-\zeta a_1 = m+x_{A,2}-\zeta a_2$, since those adblock users see ads. It is $x_{A,2} = \frac{1+\zeta a_2-\zeta a_1}{2}$.

The indifferent user among the second segment of adblock users, who remove all ads, is the one at position x_A that is the solution to the equation $m+1-x_A = m+x_A$, i.e. $x_A = \frac{1}{2}$. The expected mass of users in platform 1 who see add is $z_{1,a} = \lambda x_N + \xi x_{A,2}$. Similarly, the expected mass of users in platform 2 who see add is $z_{2,a} = \lambda (1 - x_N) + \xi (1 - x_{A,2})$.

The profit for platform 1 is $z_{1,a}a_1 - f$ and the profit for platform 2 is $z_{2,a}a_2 - f$, since they also have to pay the fee f to the ad-blocker company. Thus, to find the advertising intensities a_1, a_2 in the equilibrium, we need to solve the system $\frac{\partial(z_{1,a}a_1-f)}{\partial a_1} = \frac{\partial(z_{2,a}a_2-f)}{\partial a_2} = 0$. The solution is $a_1 = a_2 = \frac{\lambda + \xi}{\beta \lambda + \zeta \xi}$. From this, we get that the profit for both platforms is equal to $\frac{(\lambda + \xi)^2}{2(\beta \lambda + \zeta \xi)} - f$.

In Table 5, we can see the payoff matrix of the full game. An example where (WHITELIST, WHITELIST) is the unique equilibrium of the game is $\lambda = \mu = \nu = 1$, $\beta = \zeta = \frac{1}{10}$, and $\gamma = \frac{6}{5}$. In that case, the payoff matrix becomes

	Ban	Allow	${ m Fee}$	Whitelist
Ban	3.21, 3.21	1.13, 1.59	3.21, 1.5	1.54, 3.3
Allow	1.59, 1.13	5,5	3.35, 0.61	3.67, 5.1
$\mathbf{F}\mathbf{E}\mathbf{E}$	1.5, 3.21	0.61, 3.35	1.5, 1.5	0.96, 8.1
Whitelist	3.3, 1.54	5.1, 3.67	8.1, 0.96	10, 10

Proof of Proposition 5. This proof is similar to the proof of Proposition 1 with the additional step in every case of the decision about qualities by content creators. To avoid repetition, we analyze one case here and then we provide the payoff matrix of the full game.

Both platforms ALLOW ad blockers. The indifferent user among the non-adblock users is the one at position x_N that is the solution to the equation $m + 1 - x_N - \beta a_1 + rq_1 = m + x_N - \beta a_2 + rq_2$, i.e. $x_N = \frac{1+\beta(a_2-a_1)-r(q_2-q_1)}{2}$.

The indifferent user among those who use ad blockers is the one at position x_A that is the solution to the equation $m + 1 - x_A + rq_1 = m + x_A + rq_2$, since now this type of users can use ad blockers to avoid ads. It is $x_A = \frac{1 - r(q_2 - q_1)}{2}$.

The expected mass of users who see ads in platform 1 is $z_1 = \lambda x_N$, while the expected mass of users who see ads in platform 2 is $z_2 = \lambda (1 - x_N)$.

The profit for the content creators of platform 1 is $f_1 z_1 a_1 - c_1 q_1^2$, while the profit for the content creators of platform 2 is $f_2 z_2 a_2 - c_2 q_2^2$.⁶³ To find the qualities q_1 , q_2 , we need to solve the system $\frac{\partial (f_1 z_1 a_1 - c_1 q_1^2)}{\partial q_1} = \frac{\partial (f_2 z_2 a_2 - c_2 q_2^2)}{\partial q_2} = 0$. This gives the solution $q_1 = \frac{a_1 f_1 \lambda r}{4c_1}$ and $q_2 = \frac{a_2 f_2 \lambda r}{4c_2}$ as a function of the ad intensities a_1 and a_2 .

The profit for platform 1 is $(1 - f_1)z_1a_1$ and the profit for platform 2 is $(1 - f_2)z_2a_2$. Thus, to find the ad intensities a_1 , a_2 in the equilibrium, we need to solve the system $\frac{\partial((1-f_1)z_1a_1)}{\partial a_1} = \frac{\partial((1-f_2)z_2a_2)}{\partial a_2} = 0$. The solution is $a_1 = \frac{4c_1}{4\beta c_1 - f_1\lambda r^2}$ and $a_2 = \frac{4c_2}{4\beta c_2 - f_2\lambda r^2}$. From this, we get that the profit for platform 1 is equal to $\frac{2c_1(1-f_1)\lambda}{4\beta c_1 - f_1\lambda r^2}$ and the profit for platform 2 is $\frac{2c_2(1-f_2)\lambda}{4\beta c_2 - f_2\lambda r^2}$. Moreover, the qualities are $q_1 = \frac{f_1\lambda r}{4\beta c_1 - f_1\lambda r^2}$ and $q_2 = \frac{f_2\lambda r}{4\beta c_2 - f_2\lambda r^2}$.

 $^{^{63}}$ For simplicity in this analysis and to avoid corner solutions we assume that the cost parameters c_1 and c_2 are sufficiently large.

The user welfare of non-adblock users is

$$\begin{split} \lambda \left(\int_0^{\frac{1}{2}} \left(m + 1 - x - \beta \cdot \frac{4c_1}{4\beta c_1 - f_1\lambda r^2} + r \cdot \frac{f_1\lambda r}{4\beta c_1 - f_1\lambda r^2} \right) dx \\ + \int_{\frac{1}{2}}^1 \left(m + x - \beta \cdot \frac{4c_2}{4\beta c_2 - f_2\lambda r^2} + r \cdot \frac{f_2\lambda r}{4\beta c_2 - f_2\lambda r^2} \right) dx \end{split} \\ = \lambda \left(m - \frac{1}{4} \right). \end{split}$$

The user welfare of adblock users is

$$\mu \left(\int_0^{x_A} \left(m + 1 - x + r \cdot \frac{f_1 \lambda r}{4\beta c_1 - f_1 \lambda r^2} \right) dx + \int_{1 - x_A}^1 \left(m + x + r \cdot \frac{f_2 \lambda r}{4\beta c_2 - f_2 \lambda r^2} \right) dx \right)$$

where $x_A = \frac{1}{2} + \frac{2\beta c_1}{4\beta c_1 - f_1\lambda r^2} - \frac{2\beta c_2}{4\beta c_2 - f_2\lambda r^2}$. For symmetric platforms, i.e. when $f_1 = f_2 = \hat{f}$ and $c_1 = c_2 = \hat{c}$, the user welfare of adblock users is

$$\mu\left(m + \frac{3}{4} + \frac{\hat{f}\lambda r^2}{4\beta\hat{c} - \hat{f}\lambda r^2}\right) \ge \mu\left(m + \frac{3}{4}\right),$$

larger than it was in the main model.

Table 6 contains the payoff matrix of the game. The quality in the (BAN, BAN) equilibrium for platform 1 is $q_{1,\text{BAN}} = \frac{f_1(\lambda+\mu)^2 r}{4\beta c_1\lambda+4\gamma c_1\mu-f_1(\lambda+\mu)^2 r^2}$. This is less than or equal to $q_{1,\text{ALLOW}} = \frac{f_1\lambda r}{4\beta c_1-f_1\lambda r^2}$ when $\frac{\gamma}{\beta} \ge 2 + \frac{\mu}{\lambda}$, i.e. for sufficiently high $\frac{\gamma}{\beta}$. Similarly, $q_{2,\text{BAN}} \le q_{2,\text{ALLOW}}$ when $\frac{\gamma}{\beta} \ge 2 + \frac{\mu}{\lambda}$.

The quality in the (FEE, FEE) equilibrium for platform 1 is $q_{1,\text{FEE}} = \frac{f_1(\lambda+\mu)r}{4c_1-f_1(\lambda+\mu)r^2}$. This is less than or equal to $q_{1,\text{ALLOW}} = \frac{f_1\lambda r}{4\beta c_1-f_1\lambda r^2}$ when $\beta \leq \frac{\lambda}{\lambda+\mu}$, i.e. for sufficiently low β . Similarly, $q_{2,\text{FEE}} \leq q_{2,\text{ALLOW}}$ when $\beta \leq \frac{\lambda}{\lambda+\mu}$.

For the case of symmetric platforms, the total user welfare in the (BAN, BAN) and in the (FEE, FEE) equilibria is $(\lambda + \mu) \left(m - \frac{1}{4}\right)$, i.e. the same as in the main model. However, in the (ALLOW, ALLOW) equilibrium, the total user welfare is $\lambda \left(m - \frac{1}{4}\right) + \mu \left(m + \frac{3}{4} + \frac{\hat{f}\lambda r^2}{4\beta\hat{c} - \hat{f}\lambda r^2}\right) \geq \lambda \left(m - \frac{1}{4}\right) + \mu \left(m + \frac{3}{4}\right)$, larger than in the main model.

Proof of Proposition 6. Before we move to the general case, we will show an example of an equilibrium for illustrative purposes. Let's assume that B = 0.1, $\Gamma = 1.4$, $\lambda = 1$, $\mu = 2$, $\Delta = 1$, and $a^* = 1$. This means that $t_H = 1.71429$ and $t_L = 11$. We claim that for these parameter values there is an equilibrium where both platforms allow ad blockers. Their revenues in this equilibrium are $r_1 = r_2 = 5$ and their advertising intensities are $a_1 = a_2 = 10$.

To show that this is an equilibrium, we need to consider all possible deviations of each one of the platforms. Without loss of generality due to symmetry, we consider the possible deviations of platform 2. Platform 2 can either BAN ad blockers or ALLOW them, and in either one of these cases it can pick an advertising intensity in one of four regions: $[0, a^*)$, $[a^*, t_H), [t_H, t_L), \text{ or } [t_L, +\infty).$

In Figures 19 and 20, we can see how platform 2's revenue changes as its advertising intensity a_2 changes. Figure 19 is for the case platform 2 BANs ad blockers, while Figure 20 is for the case it ALLOWS them. We observe that in both cases the maximum revenue platform 2 can achieve is equal to 5 when $a_2 = 10$. Therefore, there is no profitable deviation for platform 2.



Figure 19: Platform 2's revenue as a function of the advertising intensity a_2 when platform 2 BANS ad blockers and platform 1 ALLOWS ad blockers with $a_1 = 10$, for B = 0.1, $\Gamma = 1.4$, $\lambda = 1$, $\mu = 2$, $\Delta = 1$, and $a^* = 1$.

In Figure 21, we can see the equilibria in different regions of the parameter space. When B is high or $\frac{\Gamma}{B}$ is low, the only equilibrium is for both platforms to BAN ad blockers. Otherwise, there is an equilibrium where both platforms ALLOW ad blockers. Sometimes this is the best equilibrium among the two, while sometimes (in the region in the top left of Figure 21) this is the unique equilibrium. The equilibrium we described above is an example of a unique equilibrium.

To generate Figure 21, we analyzed the general case as follows. Platforms can choose one of the following 5 possible general strategies:

- BAN₁: They ban ad blockers and they choose an advertising intensity in the interval $[0, a^*]$.
- BAN₂: They ban ad blockers and they choose an advertising intensity in the interval $[a^*, +\infty]$.
- ALLOW₁: They allow ad blockers and they choose an advertising intensity in the interval $[0, a^*]$.



Figure 20: Platform 2's revenue as a function of the advertising intensity a_2 when platform 2 ALLOWS ad blockers and platform 1 ALLOWS ad blockers with $a_1 = 10$, for B = 0.1, $\Gamma = 1.4$, $\lambda = 1$, $\mu = 2$, $\Delta = 1$, and $a^* = 1$.

- ALLOW₂: They allow ad blockers and they choose an advertising intensity in the interval $[a^*, t_H]$.
- ALLOW₃: They allow ad blockers and they choose an advertising intensity in the interval $[t_H, t_L]$.

(The strategy in which a platform allows ad blockers with advertising intensity in the interval $[t_L, +\infty]$ always gives 0 revenue to the platform, because every user will use an ad blocker. Therefore, we can ignore this strategy from the analysis.)

Since each platform can choose one of these possible strategies, there are 25 possible combinations of strategies that we need to consider.

Let's start with the case ($ALLOW_3$, $ALLOW_3$), which is the focal case. In this case, high-sensitive users will use ad blockers while low-sensitive users will choose to see ads.

Therefore, the utility from advertising of high-sensitive users will be 0 for both platforms, and the utility from advertising of low-sensitive users will be $\Delta \cdot a^* + B \cdot a^* - B \cdot a_1$ for platform 1 and $\Delta \cdot a^* + B \cdot a^* - B \cdot a_2$ for platform 2.

To find the indifferent low-sensitive user, we need to solve the equation $m + 1 - x_N + \Delta \cdot a^* + B \cdot a^* - B \cdot a_1 = m + x_N + \Delta \cdot a^* + B \cdot a^* - B \cdot a_2$, which has solution $x_N = \frac{1 + B \cdot (a_2 - a_1)}{2}$.

To find the indifferent high-sensitive user, we need to solve the equation $m+1-x_A+0 = m+x_A+0$, which has solution $x_A = \frac{1}{2}$.

The mass of users who choose to see ads in platform 1 is $z_{1,A} = x_N \cdot \lambda$ and the mass of users who choose to see ads in platform 2 is $z_{2,A} = (1 - x_N) \cdot \lambda$. The revenue for platform 1 is $r_1 = z_{1,A} \cdot a_1$ and the revenue for platform 2 is $r_2 = z_{2,A} \cdot a_1$.

For a given a_2 , r_1 is a convex function of a_1 . Therefore, there are three possible cases for what the optimal value of a_1 for platform 1 is. Let $V(a_2)$ be the solution of the equation



Figure 21: Equilibria regions $\lambda = 1$, $\mu = 2$, $\Delta = 1$, and $a^* = 1$. When there are two equilibrium strategies in the same region, the first one (in bold) is better for platforms.

 $\frac{\partial r_1}{\partial a_1} = 0$. Then we have the following:

- If $V(a_2) < t_H$, then the optimal value of a_1 is t_H .
- If $t_L \leq V(a_2) \leq t_H$, then the optimal value of a_1 is $V(a_2)$.
- If $V(a_2) > t_L$, then the optimal value of a_1 is t_L .

Similarly, for platform 2 there are three possible cases for what the optimal value of a_2 is for a given a_1 . Therefore, to find the possible equilibria we need to consider 9 subcases. Given some specific values for the parameters of the model, only one of those 9 subcases will give a valid solution which is consistent with the assumptions. We can find that solution by solving the corresponding system of equations.

For example, let's consider the cases where the valid solution is given by solving the system of equations $\frac{\partial r_1}{\partial a_1} = \frac{\partial r_2}{\partial a_2} = 0$, i.e. the cases where the solution of that system satisfies the conditions $t_L \leq a_1 \leq t_H$ and $t_L \leq a_2 \leq t_H$. The solution to this system is $a_1 = a_2 = \frac{1}{B}$. The revenues of the platforms are then $r_1 = r_2 = \frac{\lambda}{2B}$.

We can follow the same procedure for each one of the 25 cases and find the revenues of the platforms for each one of them. This will give as a 5x5 matrix similar to those in Section A.4. The only difference is that due to the thresholds a^*, t_H, t_L , the optimal values for the advertising intensities are sometimes given by corner solutions, so this matrix will look different for different values of the parameters of the model (we can think of this as 9 possible different pairs of revenues in each cell of the matrix).

For given values of the parameters, we can find the matrix and use it to find the equilibria of the model. This is how Figure 21 was generated.

If we consider the focal case again, which corresponds to the upper-left region of Figure 21, the revenues of the platforms in the (BAN₂, BAN₂) case are $r_1 = r_2 = \frac{(\lambda + \mu)^2}{2(B\lambda + \Gamma\mu)}$.

If we now compare the quantities $\frac{\lambda}{2B}$ and $\frac{(\lambda+\mu)^2}{2(B\lambda+\Gamma\mu)}$, we see the following.

- For fixed values of λ, μ, B , the second quantity decreases as $\frac{\Gamma}{B}$ increases, while the first quantity remains unchanged. In other words, for sufficiently high $\frac{\Gamma}{B}$, ALLOW is better than BAN.
- For fixed values of $\lambda, \mu, \frac{\Gamma}{B}$, the first quantity increases as *B* decreases, while the second quantity remains unchanged. In other words, for sufficiently low *B*, ALLOW is better than BAN.

The numerical example that we started with, is an example where there is is a unique equilibrium in which both platforms use the ALLOW strategy. \Box

Proof of Proposition 7. In the benchmark model, where ad blockers do not exist, every consumer will see an ad for the product and will learn his valuation v and the price of the product p. If $v \ge p$, the consumer will buy the product. Therefore, the mass of consumers who buy the product is 1-p, and the revenue for the firm is $(1-p) \cdot p$. This means that the optimal price is $p^* = \frac{1}{2}$ and the revenue for the firm in equilibrium is $r^* = \frac{1}{4}$. The consumer surplus is then $s^* = \int_0^1 \left[\int_0^{p^*} (-\sigma s) \, dv + \int_{p^*}^1 (v - p^* - \sigma s) \, dv \right] \, ds = \frac{1}{8} - \frac{\sigma}{2}$.

Now let's consider the model with ad blockers. Consider a consumer with ad sensitivity s. If he chooses to use an ad blocker, then he will not see an ad and he will not buy the product, which means that his utility will be 0. If he chooses to not use an ad blocker, then his expected utility is $-\sigma s + \int_p^1 (v-p) dv = \frac{(1-p)^2}{2} - \sigma s$, for the price p that he expects the product to cost.⁶⁴ Therefore, a consumer will decide to use an ad blocker only if $\frac{(1-p)^2}{2} - \sigma s \leq 0 \Leftrightarrow s \geq \frac{(1-p)^2}{2\sigma}$.

This means that only a mass of $\min\left\{\frac{(1-p)^2}{2\sigma}, 1\right\}$ of consumers will see an ad, and among those the probability that someone will have valuation above the price p is 1-p. Therefore, the expected revenue for the firm is $\min\left\{\frac{(1-p)^2}{2\sigma}, 1\right\} \cdot (1-p) \cdot p$. From this expression we can see that there are three cases for the optimal price \hat{p} :

$$\hat{p} = \begin{cases} \frac{1}{2} & , \text{ if } \sigma \leq \frac{1}{8}, \\ 1 - \sqrt{2\sigma} & , \text{ if } \frac{1}{8} < \sigma < \frac{9}{32} \\ \frac{1}{4} & , \text{ if } \sigma \geq \frac{9}{32}. \end{cases}$$

Note that in the first two cases none of the consumers is using an ad blocker. Also, in the last two cases the price is strictly lower compared to the benchmark.

A non-adblock user will have the same disutility from the ad as in the benchmark model and he will have to pay a lower price if he decides to buy the product. Therefore, a nonadblock user is better off.

Now let's consider a consumer who is using an ad blocker. Since a consumer like that exists, it must be $\sigma \geq \frac{9}{32}$ and $\hat{p} = \frac{1}{4}$. His ad sensitivity s is then at least $\frac{(1-\hat{p})^2}{2\sigma} := \bar{s}$ and his utility is 0. In the benchmark model, his expected utility is $\frac{1}{8} - \sigma s \leq \frac{1}{8} - \sigma \bar{s} = \frac{1}{8} - \frac{9}{32} < 0$. Therefore, an adblock user is better off as well.

⁶⁴In equilibrium this price will be the actual price the firm charges.

Proof of Proposition 8. Among those consumers who don't see add the indifferent one between searching and doing nothing is the one with ad sensitivity s_1 such that $-\psi(s_1) + \frac{(1-p)^2}{2} = 0$. Solving this we get $s_1 = \frac{(1-p)^2 - 2\phi_1}{2\phi_2}$. Among those consumers who don't search the indifferent one between seeing add and

Among those consumers who don't search the indifferent one between seeing ads and doing nothing is the one with ad sensitivity s_2 such that $-\sigma s_2 + \frac{(1-p)^2}{2} = 0$. Solving this we get $s_2 = \frac{(1-p)^2}{2\sigma}$.

Among those consumers who either see ads or search the indifferent one between seeing ads and searching is the one with ad sensitivity s_3 such that $-\sigma s_3 + \frac{(1-p)^2}{2} = -\psi(s_3) + \frac{(1-p)^2}{2}$. Solving this we get $s_3 = \frac{\phi_1}{\sigma - \phi_2}$.

Given s_1 , s_2 , and s_3 , we can find the mass of consumers z_A who choose to see ads, and the mass of consumers z_S who choose to search. It is $z_A = \max\{\min\{s_2, s_3, 1\}, 0\}$ and $z_S = 1 - \min\{\max s_1, s_3, 0, 1\}.$

The profit for the firm if it chooses to advertise is then $r_A = (z_A + z_S)(1 - p)p - wz_A$. If the firm does not advertise, its profit will be $r_{NA} = (1 - \min\{\max\{s_1, 0\}, 1\})(1 - p)p$. Therefore, the firm needs to find a price p that maximizes its profit $r = \max\{r_A, r_{NA}\}$.

In the benchmark model without ad blockers, the firm can either advertise to everyone with a profit of $r_A^{\text{bench}} = (1 - p)p - w$ or to no-one with a profit of $r_{NA}^{\text{bench}} = (1 - \min\{\max\{s_1, 0\}, 1\})(1 - p)p$. Therefore, the firm needs to find a price p that maximizes $r^{\text{bench}} = \max\{r_A^{\text{bench}}, r_{NA}^{\text{bench}}\}$.

All that remains to be done is to find a set of parameter values such that the optimal r is strictly higher that the optimal r^{bench} . We will give two examples here where this happens. In the first, the optimal action for the firm in the benchmark is to advertise to everyone. In the second, the optimal action for the firm is not to advertise at all. In both cases, the equilibrium with ad blockers will be better for the firm.

Let $\sigma = 0.3$, w = 0.21, $\phi_1 = 1$, and $\phi_2 = -0.9$. The optimal strategy for the firm in the benchmark is to advertise to everyone with a price of p = 0.5 and a revenue of $r^{\text{bench}} = 0.04$. With ad blockers, the optimal strategy is to advertise with a price of $p \approx 0.36584$. The mass of consumers who see ads is then $z_A \approx 0.670265$, while the mass of consumers who search is $z_S \approx 0.11231$. The final profit is then $r \approx 0.0408027 > r^{\text{bench}}$, despite the lower price. This is an example where both the firm and the consumers are better off with ad blockers.

Let $\sigma = 0.3$, w = 0.21, $\phi_1 = 1$, and $\phi_2 = -1$. The optimal strategy for the firm in the benchmark is to not advertise to anyone and set a price of p = 0.25. The mass of consumer who search is then ≈ 0.28125 and the revenue will be $r^{\text{bench}} \approx 0.0527344$. With ad blockers, the optimal strategy is to advertise with a price of $p \approx 0.380346$. The mass of consumers who see ads is then $z_A \approx 0.639953$, while the mass of consumers who search is $z_S \approx 0.191986$. The final profit is then $r \approx 0.0616835 > r^{\text{bench}}$. Note that here the price is higher with ad blockers, which is an example where consumers are worse off.

A.4 Payoff Matrices

	BAN	ALLOW	FEE	Ads or Fee
BAN	$\frac{(\lambda+\mu+\nu)^2}{2(\beta\lambda+\gamma\mu+\eta\nu)},\frac{(\lambda+\mu+\nu)^2}{2(\beta\lambda+\gamma\mu+\eta\nu)}$	$\frac{(3\lambda+2\mu+3\nu)^2(\beta\lambda+\gamma\mu+\eta\nu)}{2(3\beta\lambda+4\gamma\mu+3\eta\nu)^2}, C$	$rac{(\lambda+\mu+ u)^2}{2(eta\lambda+\gamma\mu+\eta u)},rac{1}{2}(\lambda+\mu+ u)$	$\frac{(\lambda+\mu+ u)^2}{2(\beta\lambda+\gamma\mu+\eta u)}, A$
Allow	$C, \frac{(3\lambda+2\mu+3\nu)^2(\beta\lambda+\gamma\mu+\eta\nu)}{2(3\beta\lambda+4\gamma\mu+3\eta\nu)^2}$	$rac{(\lambda + u)^2}{2(eta \lambda + \eta u)}, rac{(\lambda + u)^2}{2(eta \lambda + \eta u)}$	$\frac{9(\lambda+\nu)^2(\lambda+\mu+\nu)^2}{2(3\lambda+4\mu+3\nu)^2(\beta\lambda+\eta\nu)},\frac{(\lambda+\mu+\nu)(3\lambda+2\mu+3\nu)^2}{2(3\lambda+4\mu+3\nu)^2}$	$\frac{9(\lambda+\nu)^2(\mu+\nu)^2(\beta\lambda+\eta\nu)}{2(3\beta\lambda(\mu+\nu)+\eta\nu(4\mu+3\nu))^2}, B$
$\mathbf{F}\mathbf{E}\mathbf{E}$	$\frac{1}{2}(\lambda + \mu + \nu), \frac{(\lambda + \mu + \nu)^2}{2(\beta \lambda + \gamma \mu + \eta \nu)}$	$\frac{(\lambda+\mu+\nu)(3\lambda+2\mu+3\nu)^2}{2(3\lambda+4\mu+3\nu)^2},\frac{9(\lambda+\nu)^2(\lambda+\mu+\nu)^2}{2(3\lambda+4\mu+3\nu)^2(\beta\lambda+\eta\nu)}$	$\frac{1}{2}(\lambda + \mu + \nu), \frac{1}{2}(\lambda + \mu + \nu)$	$\frac{1}{2}(\lambda + \mu + \nu), \frac{\lambda + \beta(\mu + \nu)}{2\beta}$
Ads or Fee	$A, \frac{(\lambda+\mu+\nu)^2}{2(\beta\lambda+\gamma\mu+\eta\nu)}$	$B, \frac{9(\lambda+\nu)^2(\mu+\nu)^2(\beta\lambda+\eta\nu)}{2(3\beta\lambda(\mu+\nu)+\eta\nu(4\mu+3\nu))^2}$	$rac{\lambda+eta(\mu+ u)}{2eta},rac{1}{2}(\lambda+\mu+ u)$	$rac{\lambda+eta(\mu+ u)}{2eta}, rac{\lambda+eta(\mu+ u)}{2eta}$

Table 4: Payoff matrix in the model with the ADS OR FEE plan.

 $A = \frac{\lambda^2(\mu+\nu)^2\beta^3 + \lambda(\mu+\nu)\left(4\lambda^2 + 2((\gamma+2)\mu+(\eta+2)\nu)\lambda + (\mu+\nu)(4\gamma\mu+\mu+4\eta\nu+\nu)\right)\beta^2 + (\gamma\mu+\eta\nu)\left(((\gamma+4)\mu+(\eta+4)\nu)\lambda^2 + 2(\mu+\nu)(2\gamma\mu+\mu+2\eta\nu+\nu)\lambda + 4(\mu+\nu)^2(\gamma\mu+\eta\nu)\right)\beta + \lambda(\mu+\nu)(\gamma\mu+\eta\nu)^2}{8\beta(\mu+\nu)(\beta\lambda+\gamma\mu+\mu\nu)^2}$

$$B = \frac{9\beta^{3}\lambda^{2}(\mu+\nu)^{3}+3\beta^{2}\lambda(12\lambda^{2}+6(\eta+2)\nu\lambda+\nu(8\eta\mu+12\eta\nu+3\nu))(\mu+\nu)^{2}+\beta\eta\nu((48\mu+9(\eta+4)\nu)\lambda^{2}+6\nu(4(\eta+1)\mu+6\eta\nu+3\nu)\lambda+4\eta\nu(2\mu+3\nu)^{2})(\mu+\nu)+\eta^{2}\lambda\nu^{2}(4\mu+3\nu)^{2}}{8\beta(3\beta\lambda(\mu+\nu)+\eta\nu(4\mu+3\nu))^{2}}$$

$$C = \frac{(2\gamma\mu(\lambda+\nu)+\beta\lambda(3\lambda+\mu+3\nu)+\eta\nu(3\lambda+\mu+3\nu))^{2}}{2(\beta\lambda+\eta\nu)(3\beta\lambda+4\gamma\mu+3\eta\nu)^{2}}$$

	BAN	ALLOW	FEE	WHITELIST
Ban	$rac{(\lambda+\mu+\xi)^2}{2(eta\lambda+\gamma\mu+\zeta\xi)}, rac{(\lambda+\mu+\xi)^2}{2(eta\lambda+\gamma\mu+\zeta\xi)}$	$\frac{(\beta\lambda+\gamma\mu+\zeta\xi)(3\lambda+2(\mu+\xi))^2}{2(3\beta\lambda+4\gamma\mu+4\zeta\xi)^2}, E$	$rac{(\lambda+\mu+\xi)^2}{2(eta\lambda+\gamma\mu+\zeta\xi)},rac{1}{2}(\lambda+\mu+\xi)$	$\frac{(3\lambda+2\mu+3\xi)^2(\beta\lambda+\gamma\mu+\zeta\xi)}{2(3\beta\lambda+4\gamma\mu+3\zeta\xi)^2}, D$
Allow	$E, \frac{(\beta\lambda + \gamma\mu + \zeta\xi)(3\lambda + 2(\mu + \xi))^2}{2(3\beta\lambda + 4\gamma\mu + 4\zeta\xi)^2}$	$rac{\lambda}{2eta},rac{\lambda}{2eta}$	$\frac{9\lambda(\lambda+\mu+\xi)^2}{2\beta(3\lambda+4(\mu+\xi))^2},\frac{(\lambda+\mu+\xi)(3\lambda+2(\mu+\xi))^2}{2(3\lambda+4(\mu+\xi))^2}$	$\frac{\lambda(2\zeta\xi+\beta(3\lambda+\xi))^2}{2\beta(3\beta\lambda+4\zeta\xi)^2},\frac{(3\lambda+2\xi)^2(\beta\lambda+\zeta\xi)}{2(3\beta\lambda+4\zeta\xi)^2}-f$
$\mathbf{F}\mathbf{E}\mathbf{E}$	$\frac{1}{2}(\lambda + \mu + \xi), \frac{(\lambda + \mu + \xi)^2}{2(\beta \lambda + \gamma \mu + \zeta \xi)}$	$\frac{(\lambda+\mu+\xi)(3\lambda+2(\mu+\xi))^2}{2(3\lambda+4(\mu+\xi))^2}, \frac{9\lambda(\lambda+\mu+\xi)^2}{2\beta(3\lambda+4(\mu+\xi))^2}$	$\frac{1}{2}(\lambda+\mu+\xi), \frac{1}{2}(\lambda+\mu+\xi)$	$\frac{(\lambda+\mu+\xi)(3\lambda+2\mu+3\xi)^2}{2(3\lambda+4\mu+3\xi)^2}, F$
WHITELIST	$D, \frac{(3\lambda+2\mu+3\xi)^2(\beta\lambda+\gamma\mu+\zeta\xi)}{2(3\beta\lambda+4\gamma\mu+3\zeta\xi)^2}$	$\frac{(3\lambda+2\xi)^2(\beta\lambda+\zeta\xi)}{2(3\beta\lambda+4\zeta\xi)^2} - f, \frac{\lambda(2\zeta\xi+\beta(3\lambda+\xi))^2}{2\beta(3\beta\lambda+4\zeta\xi)^2}$	$F, \frac{(\lambda+\mu+\xi)(3\lambda+2\mu+3\xi)^2}{2(3\lambda+4\mu+3\xi)^2}$	$rac{(\lambda+\xi)^2}{2(eta\lambda+\zeta\xi)}-f, rac{(\lambda+\xi)^2}{2(eta\lambda+\zeta\xi)}-f$

Table 5: Payoff matrix in the model with the WHITELIST plan.

$$D = \frac{(2\gamma\mu(\lambda+\xi)+\beta\lambda(3\lambda+\mu+3\xi)+\zeta\xi(3\lambda+\mu+3\xi))^2}{2(\beta\lambda+\zeta\xi)(3\beta\lambda+4\gamma\mu+3\zeta\xi)^2} - f$$

$$E = \frac{\lambda(\beta(3\lambda+\mu+\xi)+2(\gamma\mu+\zeta\xi))^2}{2\beta(3\beta\lambda+4\gamma\mu+4\zeta\xi)^2}$$

$$F = \frac{9(\lambda+\xi)^2(\lambda+\mu+\xi)^2}{2(3\lambda+4\mu+3\xi)^2(\beta\lambda+\zeta\xi)} - f$$

	Ban	Allow	$\mathbf{F}\mathbf{E}\mathbf{E}$
Ban	$-\frac{2c_1(f_1-1)(\lambda+\mu)^2}{4c_1(\beta\lambda+\gamma\mu)-f_1r^2(\lambda+\mu)^2}, -\frac{2c_2(f_2-1)(\lambda+\mu)^2}{4c_2(\beta\lambda+\gamma\mu)-f_2r^2(\lambda+\mu)^2}$	I, J	$-\frac{2c_1(f_1-1)(\lambda+\mu)^2}{4c_1(\beta\lambda+\gamma\mu)-f_1r^2(\lambda+\mu)^2}, -\frac{2c_2(f_2-1)(\lambda+\mu)}{4c_2-f_2r^2(\lambda+\mu)}$
Allow	G,H	$\frac{2c_1(f_1-1)\lambda}{f_1r^2\lambda-4c_1\beta}, \frac{2c_2(f_2-1)\lambda}{f_2r^2\lambda-4c_2\beta}$	$-\frac{18c_1(f_1-1)\lambda(4c_1\beta-f_1r^2\lambda)(\lambda+\mu)^2}{(3f_1r^2\lambda(\lambda+\mu)-4c_1\beta(3\lambda+4\mu))^2},L$
Fee	$-\frac{2c_1(f_1-1)(\lambda+\mu)}{4c_1-f_1r^2(\lambda+\mu)},-\frac{2c_2(f_2-1)(\lambda+\mu)^2}{4c_2(\beta\lambda+\gamma\mu)-f_2r^2(\lambda+\mu)^2}$	$K_{,-}\frac{18c_{2}(f_{2}-1)\lambda(4c_{2}\beta-f_{2}r^{2}\lambda)(\lambda+\mu)^{2}}{(3f_{2}r^{2}\lambda(\lambda+\mu)-4c_{2}\beta(3\lambda+4\mu))^{2}}$	$-\frac{2c_1(f_1-1)(\lambda+\mu)}{4c_1-f_1r^2(\lambda+\mu)},-\frac{2c_2(f_2-1)(\lambda+\mu)}{4c_2-f_2r^2(\lambda+\mu)}$

Table 6: Payoff matrix in the model with the content creators.

$$\begin{split} G &= -\frac{2c_1(f_1-1)\lambda \left(4c_1\beta - f_1r^2\lambda\right) \left(-3f_2r^2(\lambda+\mu)^2 + 8c_2\gamma\mu + 4c_2\beta(3\lambda+\mu)\right)^2}{(3f_1f_2\lambda(\lambda+\mu)^2r^4 - 4(c_1f_2\beta(\lambda+\mu)(3\lambda+4\mu) + c_2f_1\lambda(3\beta\lambda-\beta\mu+4\gamma\mu))r^2 + 16c_1c_2\beta(3\beta\lambda+4\gamma\mu))^2} \\ H &= -\frac{2c_2(f_2-1)\left(3f_1r^2\lambda(\lambda+\mu) - 4c_1\beta(3\lambda+2\mu)\right)^2\left(4c_2(\beta\lambda+\gamma\mu) - f_2r^2(\lambda+\mu)^2\right)}{(f_1\lambda(3f_2r^2(\lambda+\mu)^2 + 4c_2(-3\beta\lambda+\beta\mu-4\gamma\mu))r^2 + 4c_1\beta(4c_2(3\beta\lambda+4\gamma\mu) - f_2r^2(\lambda+\mu)(3\lambda+4\mu)))^2} \\ I &= -\frac{2c_1(f_1-1)\left(3f_2r^2\lambda(\lambda+\mu) - 4c_2\beta(3\lambda+2\mu)\right)^2\left(4c_1(\beta\lambda+\gamma\mu) - f_1r^2(\lambda+\mu)^2\right)}{(f_1(\lambda+\mu)(3f_2r^2\lambda(\lambda+\mu) - 4c_2\beta(3\lambda+4\mu))r^2 + 4c_1(f_2\lambda(-3\beta\lambda+\beta\mu-4\gamma\mu)r^2 + 4c_2\beta(3\beta\lambda+4\gamma\mu)))^2} \\ J &= -\frac{2c_2(f_2-1)\lambda\left(4c_2\beta-f_2r^2\lambda\right)\left(3f_1r^2(\lambda+\mu)^2 - 4c_1(2\gamma\mu+\beta(3\lambda+\mu))\right)^2}{(f_1(\lambda+\mu)(3f_2r^2\lambda(\lambda+\mu) - 4c_2\beta(3\lambda+4\mu))r^2 + 4c_2\beta(3\lambda+4\mu))r^2} \\ K &= -\frac{2c_1(f_1-1)(\lambda+\mu)\left(3f_2r^2\lambda(\lambda+\mu) - 4c_2\beta(3\lambda+2\mu)\right)^2}{(4c_1-f_1r^2(\lambda+\mu))(3f_2r^2\lambda(\lambda+\mu) - 4c_2\beta(3\lambda+4\mu))r^2} \\ L &= -\frac{2c_2(f_2-1)(\lambda+\mu)\left(3f_1r^2\lambda(\lambda+\mu) - 4c_2\beta(3\lambda+4\mu)\right)^2}{(4c_2-f_2r^2(\lambda+\mu))(3f_1r^2\lambda(\lambda+\mu) - 4c_2\beta(3\lambda+4\mu))^2} \end{split}$$